Sol. #1

Jones matrix $\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix} \equiv M$

(i) For full transmission

$$(M)\begin{pmatrix} A \\ B \end{pmatrix} = \lambda \begin{pmatrix} A \\ B \end{pmatrix} \rightarrow \begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \lambda \begin{pmatrix} A \\ B \end{pmatrix}$$

$$\rightarrow \left.\begin{matrix} A + iB = \lambda A \\ -iA + B = \lambda B \end{matrix}\right) \rightarrow \begin{vmatrix} 1-\lambda & i \\ -i & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 - 1 = 0 \rightarrow 1 - 2\lambda + \lambda^2 - 1 = 0$$

$$\rightarrow \lambda(\lambda - 2) = 0 \quad \therefore \lambda = 0 \ \& \ 2.$$

For $\lambda = 2$,

$$\left.\begin{matrix} A + iB = 2A \\ -iA + B = 2B \end{matrix}\right) \rightarrow \left.\begin{matrix} -A + iB = 0 \\ -iA - B = 0 \end{matrix}\right) \rightarrow \begin{matrix} -A + iB = 0 \\ A - iB = 0 \end{matrix}$$

$$\therefore \underline{A = iB}$$

$$\begin{pmatrix} i \\ 1 \end{pmatrix} = i \begin{pmatrix} 1 \\ -i \end{pmatrix}$$

The Jones vector is $\underline{\begin{pmatrix} 1 \\ -i \end{pmatrix}}$.

$\hookrightarrow$ right circularly polarized!

(ii) For zero transmission,

$$\begin{pmatrix} 1 & i \\ -i & 1 \end{pmatrix}\begin{pmatrix} A \\ B \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$\rightarrow \left[\begin{matrix} A + iB = 0 \\ -iA + B = 0 \rightarrow B = iA \end{matrix}\right. \longrightarrow \underline{A = -iB} \qquad \begin{pmatrix} -i \\ 1 \end{pmatrix} = -i \underline{\underline{\begin{pmatrix} 1 \\ i \end{pmatrix}}}$$

$\therefore$ Jones vector is $\underline{\begin{pmatrix} 1 \\ +i \end{pmatrix}}$.

$\hookrightarrow$ left circularly polarized!

Sol. # 2

(i) From Eq. (2.57), $r_p = 0$ (TM mode)

$$r_p = -\frac{\tan(\theta_B - \phi)}{\tan(\theta_B + \phi)} = 0 \quad ; \ \phi = \theta_t$$

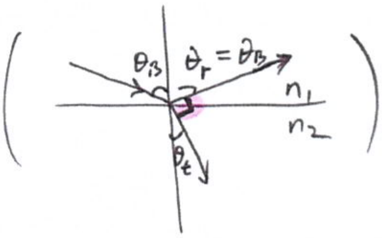
Due to Snell's law ($n_1 \sin\theta_B = n_2 \sin\phi$), $\theta \neq \phi$ due to $n_1 \neq n_2$.

$\therefore \tan(\theta_B + \phi) = \infty$ to make $r_p = 0$.

$\rightarrow \theta_B + \phi = \frac{\pi}{2}$ $\quad \therefore$ The angle btwn reflected and transmitted beams is a right angle.

(ii) From Eq. (2.59),

$$r_p = \frac{-n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}}{n^2\cos\theta + \sqrt{n^2 - \sin^2\theta}} \ ; \ n = \frac{n_2}{n_1}$$

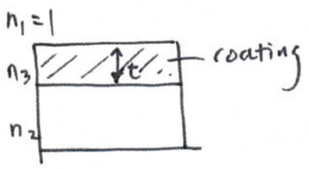


For normal incidence, $\theta = 0$.

$$\therefore r_p = \frac{-n^2 + n}{n^2 + n} = \frac{-n+1}{n+1}$$

$$R_p = |r_p|^2 = \left(\frac{-n+1}{n+1}\right)^2 = \left(\frac{n-1}{n+1}\right)^2 = \left(\frac{0.5}{2.5}\right)^2 = \underline{\underline{0.04}}$$

(iii)



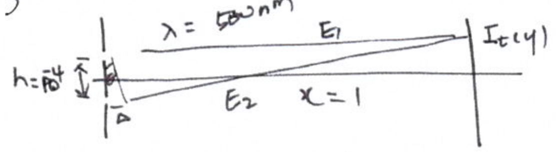
From Eq. (4.32) $\quad r = \dfrac{n_3(1-n_2)\cos kt - i(n_2 - n_3^2)\sin kt}{n_3(1+n_2)\cos kt - i(n_2 + n_3^2)\sin kt}$

for $t = \frac{\pi}{2k}$,

$\cos kt = 0$ & $\sin kt = 1$. $\quad \therefore r = \dfrac{n_2 - n_3^2}{n_2 + n_3^2} \rightarrow \underline{\underline{R = \left(\dfrac{n_2 - n_3^2}{n_2 + n_3^2}\right)^2}}$.

Sol. #3

(i)

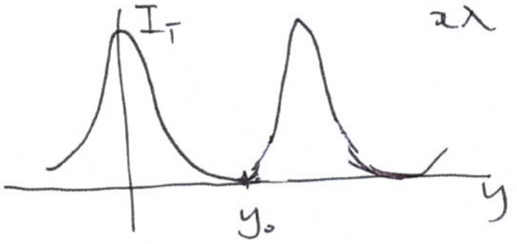


$$E_T = E_1 + E_2$$
$$= E_0 \left(1 + e^{ik\Delta}\right) e^{i(kx - \omega t)}$$
$$I_T = |E_T|^2 = I_0 \left(1 + e^{ik\Delta}\right)\left(1 + e^{-ik\Delta}\right)$$
$$= I_0 \left(1 + 1 + e^{ik\Delta} + e^{-ik\Delta}\right)$$
$$= 2 I_0 \left(1 + \cos k\Delta\right).$$
$$I_0 = |E_0|^2$$

$$\Delta = h \sin\theta \sim h \tan\theta = \frac{hy}{x}$$

$$\therefore \quad I_T(y) = 2 I_0 \left(1 + \cos \frac{khy}{x}\right)$$
$$= 2 I_0 \left[1 + \cos\left(\frac{2\pi h y}{x\lambda}\right)\right]$$

(ii) $I_T(y) = 0. \longrightarrow \dfrac{2\pi h y_0}{x\lambda} = \pi \longrightarrow y_0 = \dfrac{x}{2h}\lambda$



(iii) The energy is the concept of average in time.

$$\langle I_T \rangle = \frac{1}{Y}\int_0^Y I_T\, dy = 2 I_0, \quad \text{where} \quad \int \cos\left(\frac{2\pi y h}{x\lambda}\right) dy = 0.$$

This satisfies the energy conservation law!

Sol. # 4

(i) $R = |r|^2 = \dfrac{(n_T - n_1^2)^2}{(n_T + n_1^2)^2} = \left(\dfrac{1.52 - 1.35^2}{1.52 + 1.35^2}\right)^2 = \underline{8.2 \times 10^{-3}}$

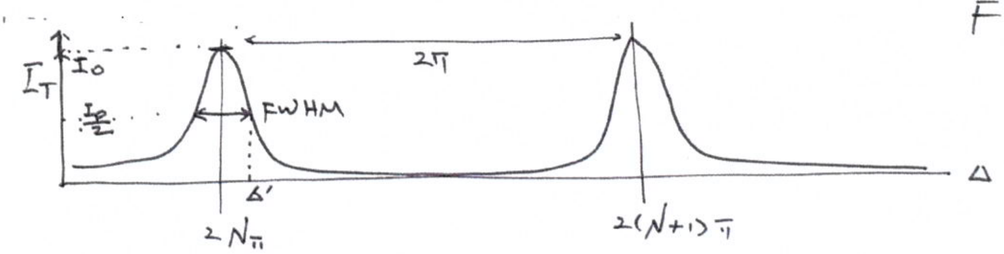
(ii) For $R = 0$, $\underline{n_T = \sqrt{n_1}}$.

Sol. #5

From Eq. (4.8)

$$I_T = I_0 \frac{T^2}{(1-R)^2} \frac{1}{1 + F\sin^2\frac{\Delta}{2}} \Rightarrow I_0 \frac{1}{1 + F\sin^2\frac{\Delta}{2}},$$

$$F = \frac{4R}{(1-R)^2}$$



For FWHM, $I_T = \frac{1}{2} I_0 \longrightarrow F\sin^2\frac{\Delta'}{2} = 1$

Because FWHM is 1% of 2π,

$\Delta' = \dfrac{\text{FWHM}}{2} = 0.01 \times 2\pi \longrightarrow \sin^2\dfrac{\Delta'}{2} = \sin^2\dfrac{(0.01 \times 2\pi)}{2}$

$= \sin^2(0.0314)$

$= \sin^2(1.8^\circ)$

$\doteqdot 0.001$

$\left(\begin{array}{l} 0.0314 : \pi = x : 180 \\ x = \dfrac{(0.0314)(180)}{\pi} = 1.8^\circ \end{array} \right.$

$\therefore F = 1000$

From $F = \dfrac{4R}{(1-R)^2} = 1000 \rightarrow (1 - 2R + R^2) \cdot 1000 = 4R$

$\rightarrow 1000R^2 - 2004R + 1000 = 0$

$\therefore R = \dfrac{2004 \pm \sqrt{2004^2 - 4000000}}{2000}$

$\doteqdot \dfrac{2004 \pm 126}{2000}$

$\therefore \underline{R = 0.939}$