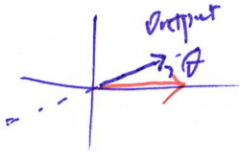
Hw#9
2021.06.02
EC5103

Prove equation (14-15) in p. 344 of the textbook:
$M = \begin{bmatrix} cos^2\theta & sin\theta cos\theta \\ sin\theta cos\theta & sin^2\theta \end{bmatrix}$.

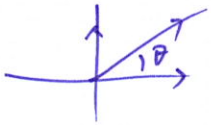
Sol) $M = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix} \stackrel{?}{=} \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

i) [output] = [M][Input].

A. for the input $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, output must be θ rotated: $\cos\theta$

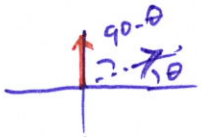


→ The decomposed ones for $\hat{x}$ & $\hat{y}$ of the θ-rotated output light: $\begin{bmatrix} \cos\theta\cos\theta \\ \cos\theta\sin\theta \end{bmatrix}$



$\Rightarrow \begin{bmatrix} \cos^2\theta \\ \cos\theta\sin\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$

B. For the input $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$, output must be 90-$\theta$ rotated: $\sin\theta$



→ The decomposed ones for $\hat{x}$ & $\hat{y}$ of the (90-θ) rotated output light $\begin{bmatrix} \sin\theta\cos\theta \\ \sin\theta\sin\theta \end{bmatrix}$

$\Rightarrow \begin{bmatrix} -\cos\theta\sin\theta \\ \sin^2\theta \end{bmatrix} = \begin{bmatrix} a & b \\ c & d \end{bmatrix}\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$,

$\therefore M = \begin{bmatrix} a & b \\ c & d \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \cos\theta\sin\theta \\ \cos\theta\sin\theta & \sin^2\theta \end{bmatrix}$. 증명.