EC5103 Quiz # 2

3/27/2017

- To satisfy equal amplitude, equal ~~ ~~ frequency condition, we assume the light as:

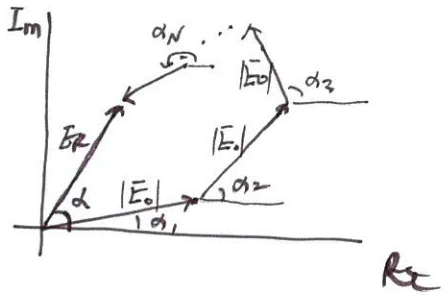
$$\bar{E}_i = E_0 e^{i(\omega t - k_j x + \phi_j)}$$

$$\rightarrow E_R = \sum_j E_0 e^{i(\omega t - k_j x + \phi_j)} = \sum_j E_0 e^{i(\omega t + \alpha_j)},$$

where $\alpha_j = \phi_j - k_j x$

(i) Using the phase diagram,

$$E_R = E_0 e^{i\omega t}\left(\sum_j \cos(\alpha_j) + i\sum_j \sin(\alpha_j)\right)$$



$$\tan\alpha = \frac{\sum_{j=1}^{N} \sin\alpha_j}{\sum_{j=1}^{N} \cos\alpha_j} \sim 0$$

due to random phase!

(ii) Intensity $I_R = |E_R|^2 = \sum_{j=1}^{N}(E_0\cos\alpha_j)^2 + \sum_{j=1}^{N}(E_0\sin\alpha_j)^2$

$$= E_0^2\sum_{j=1}^{N}\cos^2\alpha_j + E_0^2\sum_{j=1}^{N}\sin^2\alpha_j$$

$$+ 2E_0^2\sum_{j>1}^{N}\sum_{i=1}^{N}(\cos\alpha_i\cos\alpha_j + \sin\alpha_i\sin\alpha_j)$$

$$= E_0^2 \cdot N + 2E_0^2\sum_{j>i}^{N}\sum_{i=1}^{N}\cos(\alpha_j - \alpha_i)$$

$$\therefore |E_R|^2 = I_R = \boxed{N E_0^2 \text{ for random phase}}$$

$$\therefore \underline{\underline{I_R = 100 E_0^2}}$$

(iii) If the are coherent,

Let $\alpha_j = \alpha_i$

then $\boxed{I_R = N^2 E_0^2} = \underline{\underline{10^4 E_0^2}}$

(c.f.) ~~ ~~ $\left[\sum_{i=1}^{N}(A_i)\right]^2 = (A_1 + A_2 + \cdots + A_N)^2 = \sum_i A_i^2 + 2\sum_{j>i}^{N}\sum_{i=1}^{N} A_i A_j$

$$= \underline{N^2 A_0^2}, \quad \text{where } A_1 = A_2 = A_3 = \cdots = A_N = A_0.$$

iv) $E_R = E_0 \cos(\alpha - \omega t)$.

$\rightarrow \quad E_0 = E_{01} \quad ; \quad \alpha = kx + \phi_i$

① $\tan\alpha = \dfrac{\sum_{i=1}^{100} \sin\alpha_i}{\sum_{i=1}^{100} \cos\alpha_i}$ from the phase diagram in (i)

② $E_0^2 = \sum_{i=1}^{100} E_{0i}^2$ from ii)

v) $I = N I_0$ from (i)

vi) If the sources are coherent, $\rightarrow \phi_i = \phi_0 \rightarrow 0$

then the interference term in ii),

$2 \sum_{j>i}^{N} \sum_{i=1}^{N} E_{0i} E_{0j} \cos(0)$, becomes $2 \sum_{j>i}^{N} \sum_{i=1}^{N} E_{0i} E_{0j}$

Here, $E_{0i} = E_{0j} = E_{01}$ &

$2 \sum_{j>i}^{N} \sum_{i=1}^{N} E_{01} \cdot E_{01} = 2 I_0 \sum_{i=1}^{N} (i)$

From (ii) $E_R^2 = \sum_{i=1}^{N} E_{01}^2 + 2 \sum_{i=1}^{N} (i) (E_{01})^2$

$= N I_0 + 2 \dfrac{I_0 (N)(N-1)}{2}$

$= I_0 (N^2 + N - N) = I_0 N^2$.