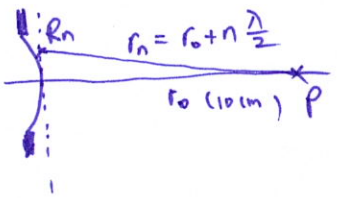
EC5103
Quiz#7 May 31, 2017
Name:

In the Fresnel zone plate the radius R_n of the n^{th} zone is denoted by $R_n = \sqrt{nr_0\lambda}$.
For λ=500 nm and the detection position P=10 cm from the zone plate,

(i) Prove $R_n = \sqrt{nr_0\lambda}$. Hint: use a spherical wave at the circular aperture.
(ii) Find out total diffraction intensity I when only even zones are open up to n=10. Assume that the light intensity with all open zones is $I_0/4$, where $I_0 = |A_1|^2$.
(iii) If P is moved forward the zone plate to be P'=5 cm, what happen to I for (i)?

Sol)



(i) $R_n^2 = (r_0 + n\frac{\lambda}{2})^2 - r_0^2$

$= r_0^2[(1 + \frac{n\lambda}{2r_0})^2 - 1]$

$= r_0^2[1 + \frac{n\lambda}{r_0} + \frac{1}{2!}(\frac{n\lambda}{2r_0})^2 \cdot 2 \cdot 1 + \cdots 0 - 1]$

$= r_0^2(n\frac{\lambda}{r_0})$, $r_0 \gg n\lambda$

$\therefore R_n = \sqrt{nr_0\lambda}$

(ii) R_2, R_4, R_6, R_8, R_{10} zones are open!

$\therefore A_{10} = a_2 + a_4 + a_6 + a_8 + a_{10} \sim 5a_1$; $a_1 \sim a_2 \sim a_{10}$

$\therefore I = (A_{10})^2 = 25 I_0$, $I_0 = a_1^2$

(iii) r_0 : 10cm → 5 cm (r_0')

from $R_n = \sqrt{nr_0\lambda}$ ⇒ $R_n' = \sqrt{n'r_0'\lambda} = \sqrt{n'(\frac{r_0}{2})\lambda} = R_n$

$\therefore n' = 2n$

This means that each zone is doubled but out of phase.

$A_2 \rightarrow A_{21} + A_{22}e^{i\pi} = 0$

$\therefore I = 0$ at $P = 5$ cm.