HW#7 Sol.

a) Show $V = \dfrac{2\sqrt{I_1 I_2}\,|\gamma(\tau)|}{(I_1 + I_2)}$

→ From (9-34),

$$V = \frac{I_{max} - I_{min}}{I_{max} + I_{min}}.$$

From (9-27), $I_p = I_{1p} + I_{2p} + 2\sqrt{I_{1p} I_{2p}}\,\mathrm{Re}(\gamma(\tau))$.

From (9-30), $\mathrm{Re}[\gamma(\tau)] = \left(1 - \frac{\tau}{\tau_0}\right)\cos\omega\tau$

$\therefore$ $\mathrm{Max}(\mathrm{Re}[\gamma(\tau)]) = 1 - \frac{\tau}{\tau_0}$

$\mathrm{Min}(\mathrm{Re}[\gamma(\tau)]) = -\left(1 - \frac{\tau}{\tau_0}\right)$.

From (9-31), $|\gamma(\tau)| = 1 - \frac{\tau}{\tau_0}$

Thus,

$$V = \frac{I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma(\tau)| - \left(I_1 + I_2 - 2\sqrt{I_1 I_2}\,|\gamma(\tau)|\right)}{I_1 + I_2 + 2\sqrt{I_1 I_2}\,|\gamma(\tau)| + I_1 + I_2 - 2\sqrt{I_1 I_2}\,|\gamma(\tau)|}$$

$$= \frac{2\sqrt{I_1 I_2}\,|\gamma(\tau)|}{I_1 + I_2}$$

For equal intensities, $I_1 = I_2$, $\underline{V_{eq} = |\gamma(\tau)|}$.

b) The final fringe visibility $V' = 0.9\,V_{eq} = 0.9\,|\gamma(\tau)|$

To satisfy this, from (a), $V = \dfrac{2\sqrt{I_1 I_2}}{I_1 + I_2}\,|\gamma(\tau)| = V'$.

$$\rightarrow \frac{4 I_1 I_2}{(I_1 + I_2)^2} = 0.81$$

$(I_1 + I_2)^2 = I_1^2 + I_2^2 + 2 I_1 I_2$

$$\rightarrow \frac{4 I_1 / I_2}{(I_1/I_2 + 1)^2} = \frac{4R}{(R+1)^2} = 0.81 \qquad \therefore \underline{\underline{R = I_1 / I_2}}$$

$\rightarrow 0.81(R+1)^2 = 4R = 0.81(R^2 + 2R + 1)$

$\rightarrow 0.81R^2 - 2.38R + 0.81 = 0$. $\qquad \cdot\ R = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$\underline{R = 2.54}$ (or 0.4)