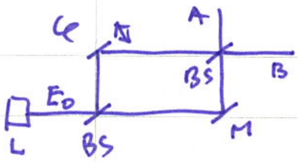
CH. 6. Quantum Possibilities and Waves

- wave : crest & trough, wavelength, period, etc.
- Interference : destructive

Go back to CH. 4 Fig. 4.8 & 4.9.

< Mathematical description >



- Matrix representation:

$$\begin{pmatrix} E_A \\ E_B \end{pmatrix} = (BS)(\varphi)(BS)\begin{pmatrix} E_0 \\ 0 \end{pmatrix}$$

$$(BS) = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}, \quad (\varphi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}$$

$$\rightarrow \begin{pmatrix} E_A \\ E_B \end{pmatrix} = \frac{E_0}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{pmatrix}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{E_0}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 & i \\ ie^{i\varphi} & e^{i\varphi} \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$= \frac{E_0}{2}\begin{pmatrix} 1-e^{i\varphi} & i(1+e^{i\varphi}) \\ i(1+e^{i\varphi}), & -(1-e^{i\varphi}) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\therefore \; E_A = \frac{E_0}{2}(1-e^{i\varphi})$$

$$E_B = \frac{iE_0}{2}(1+e^{i\varphi})$$

$$I_A = E_A E_A^* = \frac{I_0}{4}(1-e^{i\varphi})(1-e^{-i\varphi})$$

$$= \frac{I_0}{4}(1+1-2\cos\varphi)$$

$$= \frac{I_0}{2}(1-\cos\varphi)$$

$$I_B = \frac{I_0}{2}(1+\cos\varphi)$$

If $\varphi = 0$, ↙ destructive interference

$I_A = 0$, $I_B = I_0$

If $\varphi = \pi$,

$I_A = I_0$, $I_B = 0$ ↙

(∴ Same as for single photons)

< Condition for interference >

1. Photon must be in both paths.
   → But, minimum energy cannot split.
   → Thus, the particle nature must be given up.
   → The wave nature plays a role. · Superposition of probability amplitude.
2. photon path information (which way) ~~unknown~~ should not be known.
   → No which way information.
   → Indistinguishable photon characteristics.

3. Single photon self-interference (quantum) leads to cw interference (classical)

< phase φ vs. path-length difference ΔL >

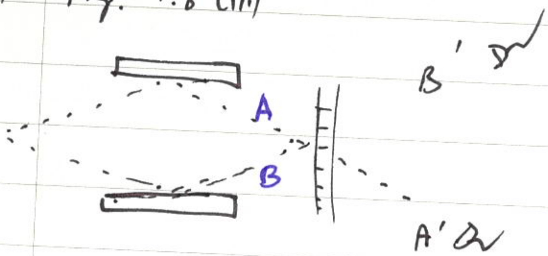
· Wave equation $e^{i(kx-\omega t)} = e^{i2\pi\left(\frac{x}{\lambda} - ft\right)}$, where $2\pi\left(\frac{x}{\lambda} - ft\right) = \varphi$

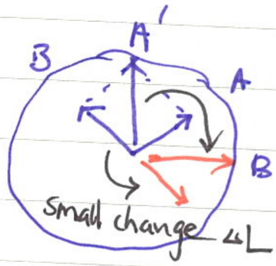
· $\Delta\varphi = \frac{2\pi}{\lambda}\Delta x = \frac{2\pi}{\lambda}\Delta L$

or $2\pi f \Delta t$ $(\lambda f = c)$

∵ BS does not alter the wavelength λ.

From Fig. 4.8 (iii)





- Small change flips the B arrow to B', resulting in the outcome at B' from A'.
- $\varphi = 2\pi \frac{\Delta L}{\lambda}$

- How small?

  → in the order of λ.

  (See also Fig. 4.2).

Q. ~~that~~ What if we block one path?
→ Both

Q. If there is no block?
→ Goat only

- Quantum path interference

Q. What if the peg slightly move up?
→ Car

Q. Why does ~~not~~ it happen in the pinball case?
→ not probability — amplitude superposition

- Quantum possibility
  - waving as a wave.

Q. What does this mean?

A. Based on the particle nature.
Without phase information. due to uncertainty principle: $\Delta n \Delta \varphi \geq 1$

→ Probability amplitude. : $|\psi\rangle$
Superposition btwn two possible paths. (bases)

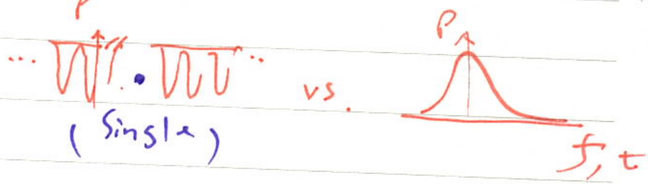
→ interference fringes

- Quantum ruler

de Broglie $\lambda = \frac{h}{p}$ (물질파)

- Schrödinger

$\psi(r,t)$: represents not a single monochromatic particle
but for information in a form of quantum possibilities → ensemble
bandwidth



Q. Does a monochromatic wave have information?

→ see Superluminal light (2000)

· Planck's Time - energy relation

· $\Delta t = \frac{h}{\Delta E}$ ← Planck's const. (6.6×10^{-34})

ΔE ← particle's enenergy

↑ cycle of internal clock

$f = \frac{1}{\Delta t} = \frac{\Delta E}{h}$. ($E = hf$).

· de Broglie ; $\lambda = \frac{h}{p}$

· E relates to velocity (matter) : $\frac{1}{2}mv^2$

frequency (photon)

· Momentum (p) : $= mv$

— inertia (관성) (tendency of not changing)

· Force (F) : $= \frac{dp}{dt} = ma$

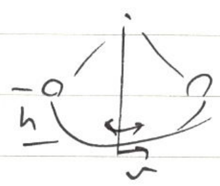
Classical physics : Trajectory (perfectly estimated)

Quantum physics : " (random : unitary)

see p. 85

· Energy (E) : $= K + V$ (K ← kinetic, V ← potential)

$\frac{1}{2}mv^2$ mgh



· Schrödinger's Wave Equation

($\psi \rightarrow e^{i\omega t}$)

($\hbar = \frac{h}{2\pi}$)

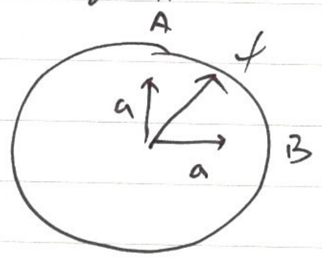
$$i\hbar\dot{\psi} = -\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$$

($i\hbar\dot{\psi}$: hf; $-\frac{\hbar^2}{2m}\nabla^2\psi + V\psi$: E)

(see p. 41) p. 122

- Born's rule : probability of finding (measurement outcome)
  $= \langle \psi | \psi \rangle = |\psi|^2$.
  (square of the quantum possibility).

- From Fig. 4.8



- Born's rule:
  $|a|^2 + |b|^2 = |\psi|^2$.
  probability

- for infinite number of possible outcomes.
  - $|\psi\rangle = \sum_{j=1}^{\infty} c_j |\psi_j\rangle$: linear superposition

  (infinite number of quantum possibilities)

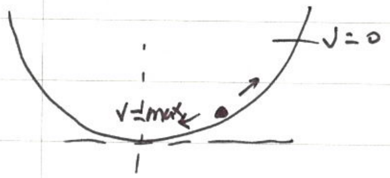
- Localization : spatial domain of finding the electron
  ↓ ↓
- Fourier series vs. Information

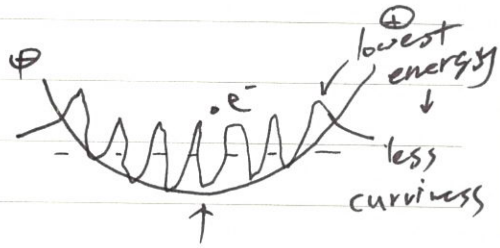
- Superposition of many (infinite) monochromatic waves

HW #4

1. $E_j = E_0 e^{i f_0 t}$, $f_0 = 10^{14}$ Hz
   For $-\Delta < \delta f_j < \Delta$, $\Delta = f_0/10$
   Simulate $E = \sum_j E_j$ and $I = EE^*$.
   ~~as functions of time (and detuning δf for E)~~ real E.
2. Bonus, weight with Gaussian function for E_j

· In a potential well (wall), See Fig. 6.4





Finding probability of mass m

(i) Classical : Max at sides
Min at Center , no interference fringe.

(ii) Quantum : Interference fringes due to backward electron, resulting in Standing a wave

$$E_s = e^{i(kx-wt)} + e^{i(kx+wt)}$$

$$= e^{ikx}\left(e^{iwt} + e^{-iwt}\right) = 2e^{ikx}\cos wt$$

or

$$E_s = e^{i(kx-wt)} + e^{i(-kx-wt)}$$

$$= e^{-iwt}\left(e^{ikx} + e^{-ikx}\right) = 2e^{-iwt}\cos kx$$

For minimum (destructive),

$$kx = \pm\left(\frac{2m+1}{2}\right)\pi, \quad m = 0, 1, 2, \dots$$

$$= \pm\frac{\pi}{2}, \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \dots$$

· Heisenberg's Uncertainty principle

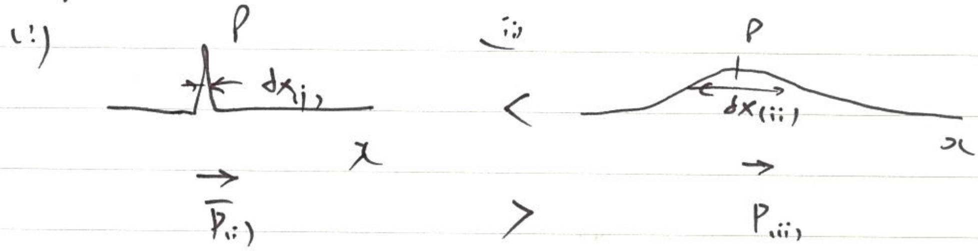
Def. A quantum particle cannot be measured precisely for both position and momentum, simultaneously.

In textbook (P.127 , middle)

... quantum particle does not have a position or momentum before it is measured.

→ Quantum particle's position and momentum are undetermined before measured.

· Let's Discuss Fig. 6.5



For the spatial spred dx,

the narrower dx, the stronger dp (velocity)

HW# 4 related interpretation,

For smaller dx, larger freq components are needed (in superposition)

→ larger energy distribution

→ larger dp.

• Uncertainty : direct consequency of nature of waves

Q. Is an electron both a particle and a wave?

A. Absolutely no because they are exclusive.

Q. Is an electron a particle sometimes and vice versa?

A. No, an electron can behave particle-like sometimes, and vice versa.

If you agree a localization with many superposed waves, then electron can be interpreted with them, superposed waves.

↓

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In the quantum theory,
both quantum and classical analyses of the first-order intensity correlation, i.e., interference fringes are identical.

↓

Born rule test

(phys. Rev. Lett. 126, 190401 (21))