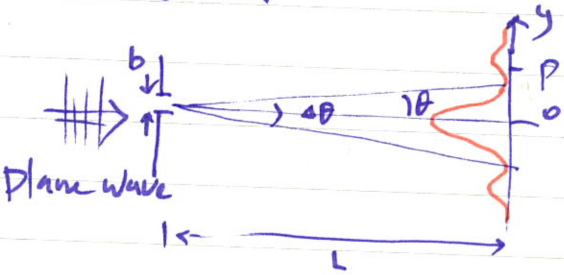
<Resolution>

- Diffraction limit

① · Single slit



$$\beta = \frac{1}{2} k b \sin\theta$$

· $E_p = E_0 \, \mathrm{sinc}\,\beta \, e^{i(kr_0 - \omega t)}$

$I_p = I_0 \, \mathrm{sinc}^2(\beta)$

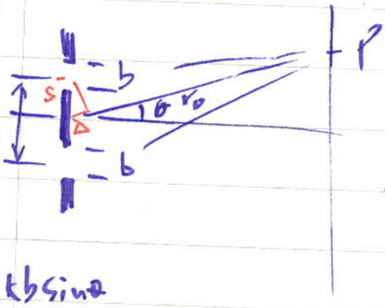
For minima, $\sin\beta = 0 \rightarrow \beta = m\pi$, $m = 0, \pm1, \pm2, \cdots$

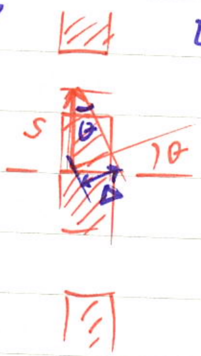
$= \frac{1}{2} k b \sin\theta_m$

$= \frac{\pi b \sin\theta_m}{\lambda}$

$$\begin{cases} y_{min} = L \sin\theta_m = \dfrac{m\lambda L}{b}. \\ L_{min} = \dfrac{b^2}{\lambda}, \text{ where } L \geq b. \\ \theta_{min} = 2\sin\theta_m = \dfrac{2\lambda}{b} \end{cases}$$

② · Double slit





$$E_p = \frac{E_L}{r_0} e^{i(kr_0 - \omega t)} \int_{s_1}^{s_2} e^{i\Delta} ds$$

$\Delta = ks\sin\theta$

$\rightarrow \frac{\sin\beta}{\beta} \cos\alpha$

· $\beta = \frac{1}{2} k b \sin\theta$

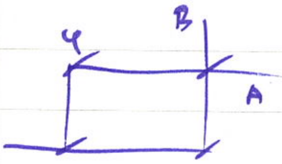
$\alpha = \frac{1}{2} k a \sin\theta$

$$\therefore I_p = I_0 \underbrace{\mathrm{sinc}^2(\beta)}_{\text{회절}} \underbrace{\cos^2(\alpha)}_{\text{간섭}} \cdots$$

$$\boxed{\cdot \text{ Airy disk}, \quad \theta = 1.22 \frac{\lambda}{b} \leftarrow (\text{diameter})}$$

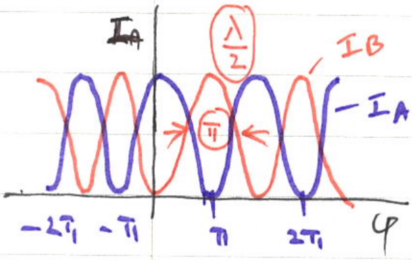
③

· MZI



$$\begin{cases} I_A = \frac{I_0}{2}(1+\cos\varphi) \\ I_B = \frac{I_0}{2}(1-\cos\varphi) \end{cases}$$

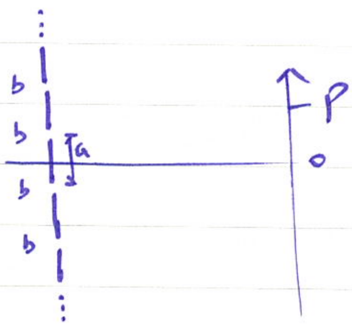
$$\cos\varphi = \cos\left(\frac{\varphi}{2}+\frac{\varphi}{2}\right) = \cos^2\frac{\varphi}{2} - \sin^2\frac{\varphi}{2}$$
$$= 2\cos^2\frac{\varphi}{2} - 1$$
$$= 1 - 2\sin^2\frac{\varphi}{2}$$



$$\therefore \begin{cases} I_A = I_0\cos^2\frac{\varphi}{2} \\ I_B = I_0\sin^2\frac{\varphi}{2} \end{cases}$$

④

· Many sits

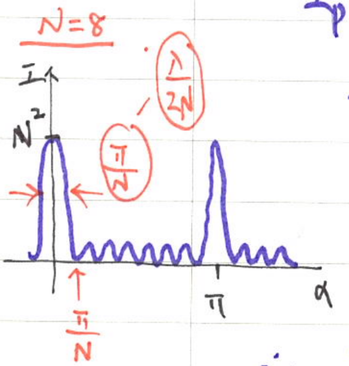


· N: slit #

$$\begin{cases} \alpha = \frac{1}{2}ka\sin\theta \\ \beta = \frac{1}{2}kb\sin\theta \end{cases}$$

$$E_p = E_0\,\mathrm{sinc}(\beta)\left(\frac{\sin N\alpha}{\sin\alpha}\right)$$

$$I_p = I_0\,\mathrm{sinc}^2(\beta)\left(\frac{\sin N\alpha}{\sin\alpha}\right)^2$$



By L'Hopital's rule, $\lim_{x\to c}\left(\frac{g(x)}{f(x)}\right) = \lim_{x\to c}\left(\frac{g'(x)}{f'(x)}\right)$

$$\lim_{\alpha\to m\pi}\frac{\sin N\alpha}{\sin\alpha} = \lim_{\alpha\to m\pi}\frac{N\cos N\alpha}{\cos\alpha} = N.$$

$$\therefore\ I_p = I_0\,\mathrm{sinc}^2(\beta)\,N^2 \quad \text{at } \alpha = m\pi$$

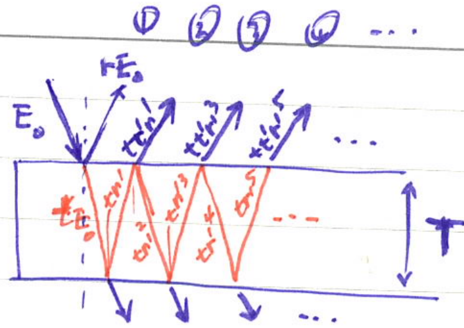
· For minimum, $\sin N\alpha = 0$. $\therefore N\alpha = p\pi$, $p = 0, \pm1, \pm2\cdots$

$$\alpha = \frac{p\pi}{N}$$

⑤

· Fabry-Perot

$\left(\sum_{j}^{N} E_j \right)$



· r: reflection coefficient

· t: transmission coefficient

· For reflected fields,

$$E_1 = rE_0\, e^{i\omega t}$$

$$E_2 = (tt'r'E_0)\, e^{i\omega t} e^{-i\delta}, \quad \delta = k\underset{\text{path length}}{\Delta} = k\, 2nT\cos\theta_t$$

$$E_3 = (tt'r'^3 E_0)\, e^{i\omega t} e^{-i2\delta}$$

$$E_4 = (tt'r'^5 E_0)\, e^{i\omega t} e^{-i3\delta}$$

$$\vdots$$

$$E_R = \sum_{i=1}^{N} E_i = E_0 e^{i\omega t}\left(r + tt'r' \sum_{i=2}^{N} \underbrace{\underbrace{r'^{(2i-4)} e^{-i(i-2)\delta}}_{x\ (= r'^2 e^{-i\delta})}}_{x^0 + x^1 + x^2 + \cdots\ =\ \frac{1}{1-x},\ |x|<1} \right)$$

$$= E_0 e^{i\omega t}\left(r + \frac{tt'r'e^{-i\delta}}{1 - r'^2 e^{-i\delta}} \right)$$

By Stokes's thm, $tt' = 1 - r^2$, $\overset{\pi}{\downarrow}\ -r' = r$

$$E_R = E_0 e^{i\omega t}\left(\frac{r - \cancel{r^3 e^{-i\delta}} - re^{-i\delta} + \cancel{r^3 e^{-i\delta}}}{1 - r^2 e^{-i\delta}} \right)$$

$$= E_0 e^{i\omega t}\left(\frac{r(1 - e^{-i\delta})}{1 - r^2 e^{-i\delta}} \right)$$

$$\therefore\ I_R = I_0 r^2 \left(\frac{1 - e^{-i\delta}}{1 - r^2 e^{-i\delta}} \right)\left(\frac{1 - e^{i\delta}}{1 - r^2 e^{i\delta}} \right)$$

$$= I_0 \frac{2r^2(1 - \cos\delta)}{1 - 2r^2\cos\delta + r^4}$$

Likewise,

$$I_T = I_0 \frac{(1 - r^2)^2}{1 - 2r^2\cos\delta + r^4}$$

(i) For $\delta = (2m-1)\pi$, $m = 1, 2, 3, \dots$

$\cos\delta = -1$.

$$\therefore\ I_R = I_0 \frac{4r^2}{(1+r^2)^2} \quad \& \quad I_T = I_0 \frac{(1-r^2)^2}{(1+r^2)^2}$$

if $r \sim 1$, $I_R = I_0$ & $I_T = 0$

(ii) For $\delta = 2m\pi$, $\cos\delta = 1$.

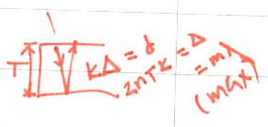
$$I_R = \underbrace{\frac{0}{(1-r^2)^2}}_{\text{zero reflection}} = 0 \quad \& \quad I_T = \underbrace{\frac{(1-r^2)^2}{(1-r^2)^2} I_0 = I_0}_{\text{full transmission}}.$$

Let $F = \frac{4r^2}{(1-r^2)^2}$ & $T = \frac{I_T}{I_0} = \frac{(1-r^2)^2}{1-2r^2\cos\delta + r^4}$

→ Coefficient of Finesse

Using $\cos\delta = \cos\left(\frac{\delta}{2}+\frac{\delta}{2}\right) = \cos^2\frac{\delta}{2} - \sin^2\frac{\delta}{2}$

$= 1 - 2\sin^2\frac{\delta}{2}$



$$1 - 2r^2\cos\delta + r^4 = 1 - 2r^2\left(1 - 2\sin^2\frac{\delta}{2}\right) + r^4$$

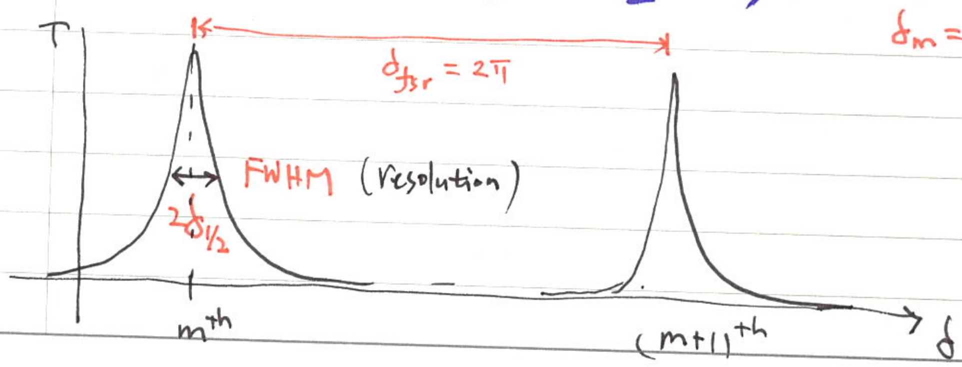
$$= (1-r^2)^2 + 4r^2\sin^2\frac{\delta}{2}.$$

$$\therefore\ T = \frac{1}{1 + F\sin^2\frac{\delta}{2}},$$

$F \gg 1$

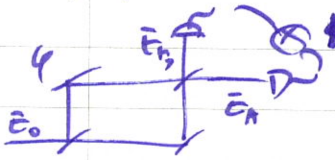
Max at $\frac{\delta}{2} = m\pi$

$\delta_m = 2m\pi$

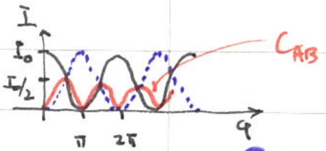


(6)

· Intensity product



$C_{AB} = I_A I_B$



· $I_A = \frac{I_0}{2}(1+\cos\varphi) = I_0 \cos^2\frac{\varphi}{2}$

$I_B = \frac{I_0}{2}(1-\cos\varphi) = I_0 \sin^2\frac{\varphi}{2}$

· $C_{AB} = I_A I_B = \frac{I_0^2}{4}(1-\cos^2\varphi) = \frac{I_0^2}{4}\sin^2\varphi$

$\cos 2\varphi = \cos^2\varphi - \sin^2\varphi = 1 - 2\sin^2\varphi$

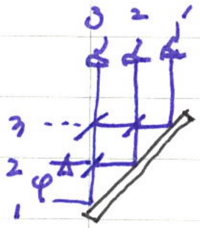
$\rightarrow C_{AB} = \frac{I_0^2}{8}(1-\cos 2\varphi)$

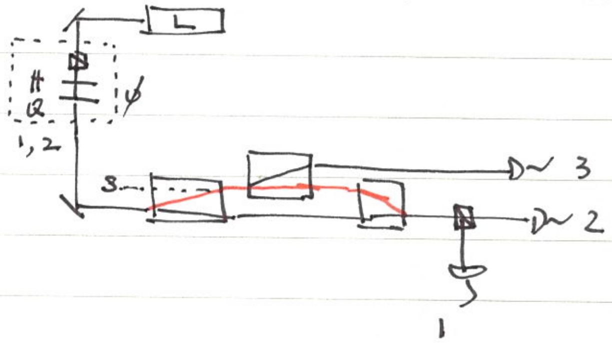
$\downarrow$?

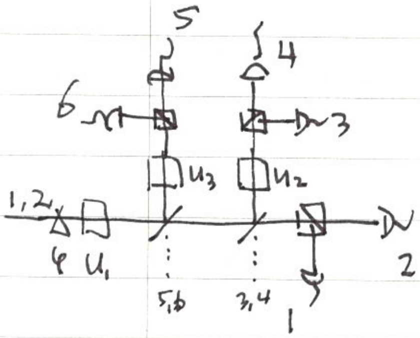
$(1-\cos N\varphi)$

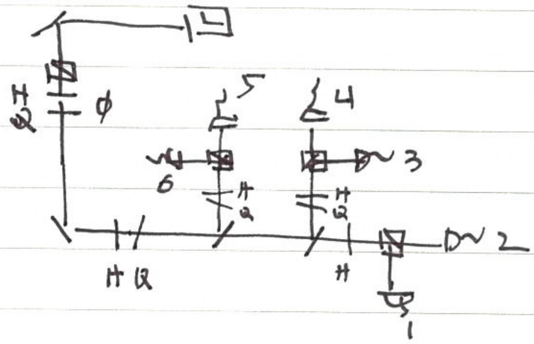
(7)

Input mode control & output mode coincidence detection









참고. PRL 98, 223601 ('07)

8 HAM's way

arXiv: 2310.13217 (2023)

- Quantum eraser
- phase control by QWP

$$\langle I_1\rangle = \langle I_3\rangle = \langle I_3'\rangle = \frac{I_0}{16}\left(1-\sin 2\theta \cos\varphi\right)$$

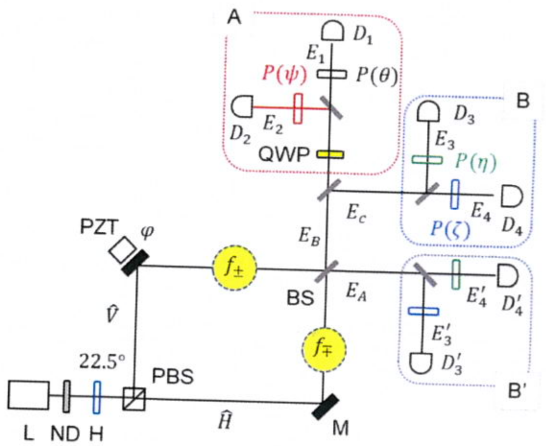
$$\langle I_2\rangle = \langle I_4\rangle = \langle I_4'\rangle = \frac{I_0}{16}\left(1+\sin 2\theta \cos\varphi\right)$$

$$\langle I_{1Q}\rangle = \frac{I_0}{16}\left(1+\sin 2\theta \sin\varphi\right)$$

$$\langle I_{2Q}\rangle = \frac{I_0}{16}\left(1-\sin 2\theta \sin\varphi\right)$$

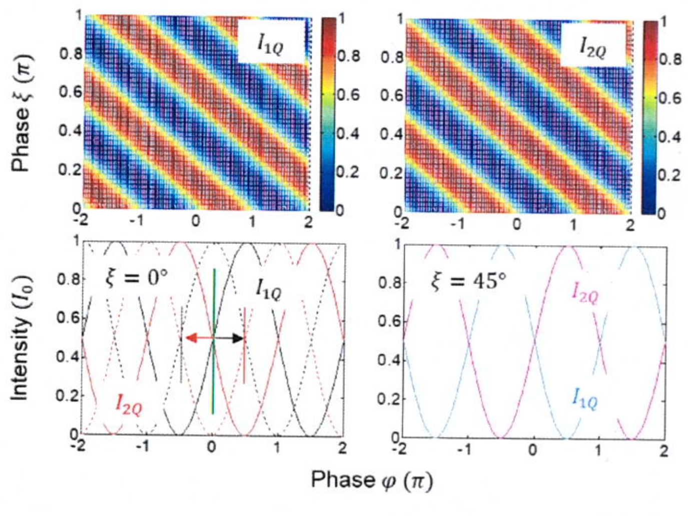
For $\theta = 45^\circ$, without QWP,

(i)
$$c_{AB} = I_A I_B = \frac{I_0^2}{256}\sin^2\varphi.$$

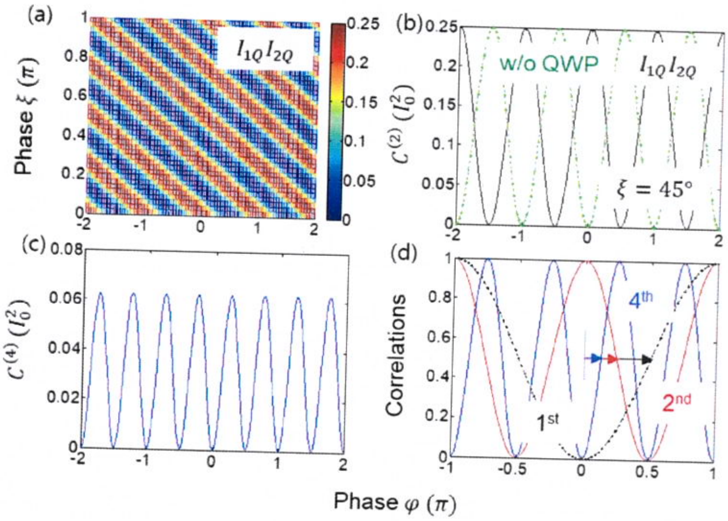


(ii) with QWP

$$C_{1Q2Q}(0) = \frac{I_0^2}{64}\cos^2\varphi$$



Ref. arXiv: 2310.13277 ('23)



Ref. PRL 98, 223601 ('07)

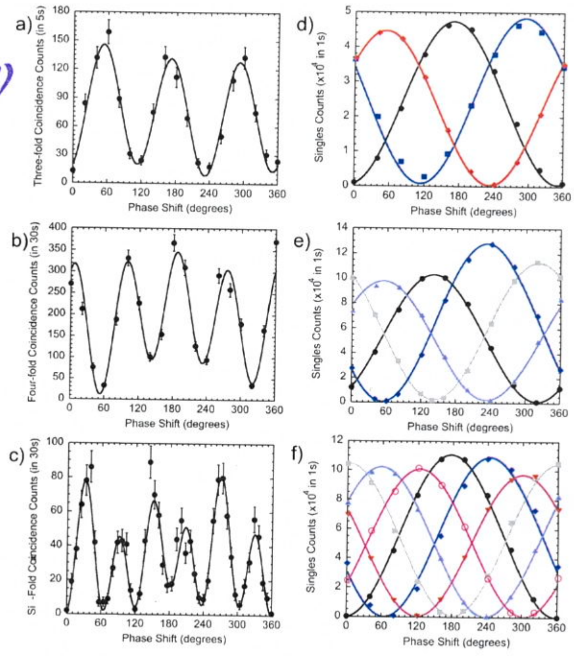


FIG. 3 (color online). (a)–(c) N-fold coincidence rates as a function of phase, ϕ, respectively, exhibiting 3, 4, and 6 distinct oscillations within a single phase cycle. The main source of uncertainty is Poissonian statistics: error bars represent the square root of the count rate. The solid line is a fit to a product of N sinusoidal fringes, as explained in the text. (d)–(f) Corresponding singles rates, each exhibiting only one oscillation per phase cycle. Error bars are contained within the data points; solid lines are the individual sinusoidal fringes obtained from (a)–(c). In (d) D1 to D3 are, respectively, indicated by black, blue, and red; in (e)–(f) D1 to D6 are indicated by black, gray, blue, cyan, red, and pink. Ideally, the phase differences between adjacent fringes is $2\pi/N$, we find: $N = 3$, $\{122°, 119°\}$; $N = 4$, $\{92°, 90°, 90°\}$; and $N = 6$, $\{55°, 66°, 56°, 62°, 56°\}$.