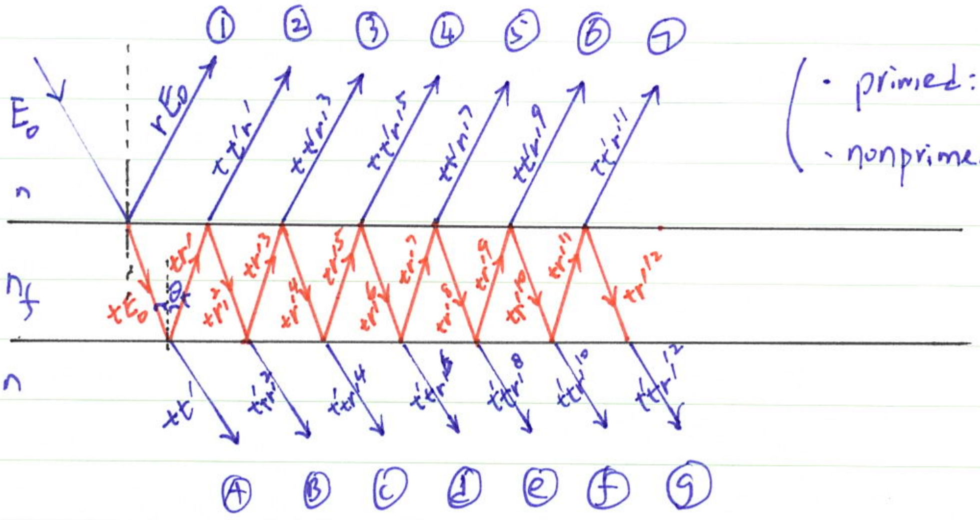
7-9 Multiple-beam Interference in a parallel plate



(· primed: $n_f \to n$
· nonprimed: $n \to n_f$)

(i) Reflected light beams :

$$E_1 = (rE_0)\, e^{i\omega t}$$
$$E_2 = (tt'r'E_0)\, e^{i\omega t} \cdot e^{-i\delta}$$
$$E_3 = (tt'r'^3 E_0)\, e^{i\omega t} \cdot e^{-i2\delta}$$
$$E_4 = (tt'r'^5 E_0)\, e^{i\omega t} \cdot e^{-i3\delta}$$
$$E_5 = (tt'r'^7 E_0)\, e^{i\omega t} \cdot e^{-i4\delta}$$
$$E_6 = (tt'r'^9 E_0)\, e^{i\omega t} \cdot e^{-i5\delta}$$
$$E_7 = (tt'r'^{11} E_0)\, e^{i\omega t} \cdot e^{-i6\delta}$$
$$\vdots$$

($\delta = k\Delta$: phase diff.
$\Delta = 2n_f t \cos\theta_t$ (7-35))

$$E_R = \sum_{i=1}^{N} E_i = E_0 e^{i\omega t}\left[r + rt'r' \underbrace{\sum_{i=2}^{\infty} r'^{(2i-4)} e^{-i(i-2)\delta}}_{1 + x + x^2 + x^3 + \cdots} \right]$$

for $x = r'^2 e^{-i\delta}$.

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(5): $1+x+x^2+x^3+\cdots \Rightarrow \frac{1}{1-x}$ for $|x|<1$

$$\therefore E_R = E_0 e^{i\omega t}\left(r + \frac{tt' r' e^{-i\delta}}{1-r'^2 e^{-i\delta}} \right)$$

Now from Stoke's relation,

$$tt' = 1-r^2 \quad ; \quad r' = -r$$

Then, $E_R = E_0 e^{i\omega t}\left(r - \frac{(1-r^2) r e^{-i\delta}}{1-r^2 e^{-i\delta}} \right)$

$$= E_0 e^{i\omega t}\left(\frac{r - \cancel{r^3 e^{-i\delta}} - r e^{-i\delta} + \cancel{r^3 e^{-i\delta}}}{1-r^2 e^{-i\delta}} \right)$$

$$= E_0 e^{i\omega t}\left(\frac{r(1-e^{-i\delta})}{1-r^2 e^{-i\delta}} \right).$$

Intensity $I \propto |E_R|^2 = E_R \cdot E_R^*$,

$$|E_R|^2 = E_0^2 r^2 \left(\frac{1-e^{-i\delta}}{1-r^2 e^{-i\delta}} \right)\left(\frac{1-e^{i\delta}}{1-r^2 e^{i\delta}} \right)$$

$$= E_0^2 r^2 \left(\frac{1-e^{i\delta}-e^{-i\delta}+1}{1-r^2 e^{i\delta} - r^2 e^{-i\delta} + r^4} \right)$$

Here $e^{i\delta}+e^{-i\delta} = 2\cos\delta$.

Thus, $I_R = \frac{2r^2(1-\cos\delta)}{1+r^4-2r^2\cos\delta} I_i \quad ; \quad I_i \propto E_0^2$

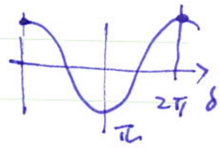
HW #5 (i) Derive I_T, (ii) Simulate I_T for $r = 0, 0.2, \cdots 1.0$ as a function of δ, $\delta = (0, 4\pi)$.

Likewise,

$$I_T = \left[\frac{(1-r^2)^2}{\underbrace{1+r^4-2r^2\cos\delta}} \right] I_i$$

Interpretation :

1. Maxima : $\delta = 2m\pi$, $m = 0, \pm1, \pm2, \ldots$



If $\delta = 2m\pi$,

$$I_T = \left(\frac{(1-r^2)^2}{(1-r^2)^2} \right) I_i = I_i \quad \text{regardless of } r.$$

2. Minima :

If $\delta = (2m-1)\pi$, $m = 1, 2, 3, \ldots$

$$I_T = \left(\frac{(1-r^2)^2}{(1+r^2)^2} \right) I_i$$

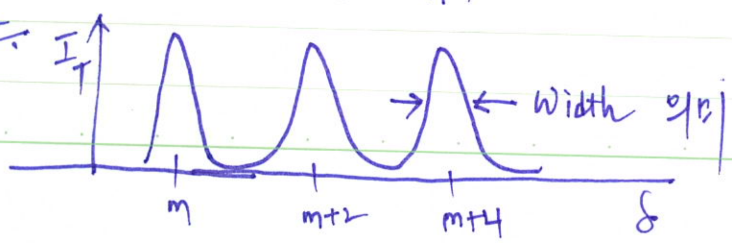
$\rightarrow$ depends on r.

ex) For $r = 0.99$,

$$I_T = I_i \quad \text{for } \delta = 2m\pi$$

$$I_T = \frac{(1-0.99^2)^2}{(1+0.99^2)^2} I_i \doteqdot 0.04\ \% \text{ of } I_i$$

Discussion.



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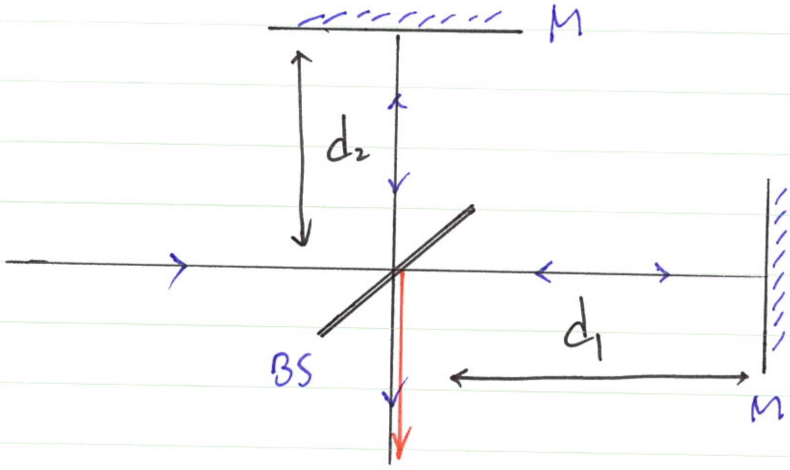
CH.8 Optical Interferometry

8-1 Michelson Interferometer



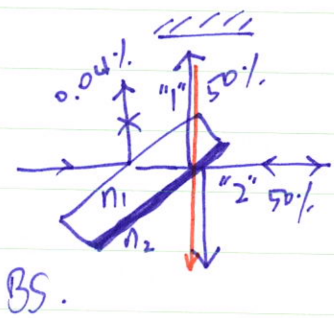
$$\Delta d = |d_1 - d_2| \cdot 2$$

· Constructive interference : ?

if $\Delta d = m\lambda$, $m = 0, 1, 2, \ldots$

→ Destructive! if $n_2 < n_1$

why



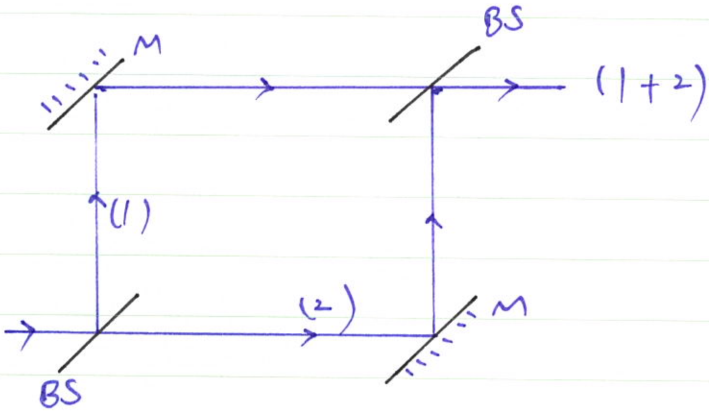
$n_1 < n_2$ or $\underline{n_1 > n_2}$

① For path "1", no phase shift.

② For path "2", π phase shift

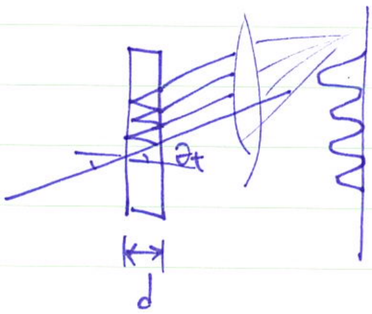
* (MgF_2 : 1.37 (n_2)
Fused Silica : 1.45 , BK7 : 1.51 (n_1)

8-3 Mach-Zehnder Interferometer



8-4 Fabry-Perot Interferometer

· etalon : fixed thickness Fabry-Perot



· $2d\cos\theta_t = m\lambda$: bright fringes

from (7-49),

$$I_T = \left(\frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta} \right) I_i$$

Transmission T,

$$T = \frac{I_T}{I_i} = \frac{(1-r^2)^2}{1+r^4-2r^2\cos\delta}$$

Here,

$$\cos\delta = 1 - 2\sin^2\left(\frac{\delta}{2}\right)$$

$$\therefore T = \frac{(1-r^2)^2}{1+r^4-2r^2\left(1-2\sin^2\frac{\delta}{2}\right)} = \frac{(1-r^2)^2}{(1-r^2)^2+4r^2\sin^2\frac{\delta}{2}}$$

$$= \frac{1}{1+\frac{4r^2\sin^2\frac{\delta}{2}}{(1-r^2)^2}}$$

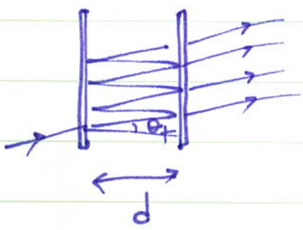
$$= \frac{1}{1+F\sin^2\frac{\delta}{2}}, \quad F \equiv \frac{4r^2}{(1-r^2)^2}$$

→ Coefficient of Finesse

Case 1 : thin slab as shown in ch. 7

$\delta = k\Delta$, $\Delta = 2n_f t\cos\theta_t$, t: thickness of the slab

Case 2 : parallel plate



$$\delta = k\Delta, \quad \Delta = 2d\cos\theta_t$$