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"Introduction to Modern Optics" by G.R. Fowles
2nd Ed.

CH. 1 The propagation of Light

1.1.

Q. What is light?
How to define it?

A. particle : Newton, Einstein, ... $\xrightarrow{20C}$ photon (Q.M.)
wave : Maxwell, Huygens, ... → coherence optics

↓ 19C ↘

E/M field Ray Optics (R, T)

(20C) → $\boxed{\text{Quantum optics}}$

· Dual nature
(complementarity, Bohr)

· Quantization

· Scrödinger eq.

$\boxed{\text{Q. electrodynamics}}$

↘ (Light-matter interaction)

· Optical processing
· Quantum Information
· Nano. photonics
· Plasmonics

↓

(Quantum A.I.)

1.2 Electrical const. & Speed of light

· Maxwell's equations in vacuum :

$$\begin{cases} \nabla \times \bar{E} = -\mu_0 \dfrac{\partial \bar{H}}{\partial t} \\ \nabla \times \bar{H} = \epsilon_0 \dfrac{\partial \bar{E}}{\partial t} \end{cases} \qquad \begin{cases} \nabla \cdot \bar{E} = 0 \\ \nabla \cdot \bar{H} = 0 \end{cases}$$

- μ_0 : magnetic permeability ($= 4\pi \times 10^{-7}$ H/m)
- ϵ_0 : electric permittivity ($= 8.85 \times 10^{-12}$ F/m)

(Wave equation)

(i) $\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E = -\nabla^2 E$

$$\Rightarrow -\mu_0 \frac{\partial}{\partial t}(\nabla \times \bar{H}) = -\epsilon_0\mu_0 \frac{\partial^2 E}{\partial t^2}$$

(ii) $\nabla \times (\nabla \times \bar{H}) = \nabla(\nabla \cdot H) - \nabla^2 H = -\nabla^2 H$

$$\Rightarrow \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \bar{E}) = -\epsilon_0\mu_0 \frac{\partial^2 H}{\partial t^2}$$

$$\rightarrow \nabla^2(\ ) = \frac{1}{c^2}\frac{\partial^2}{\partial t^2}(\ ),$$

$$c = \frac{1}{\sqrt{\epsilon_0\mu_0}} \sim 3 \cdot 10^8 \text{ (m/s)}$$

Q. What is the form of ( ) ?

· Traveling wave

$$f(z,t) \rightarrow f(z - vt, 0)$$

General solution : $f = f_0 e^{i(kz - \omega t + \phi)}$

$\rightarrow c = \dfrac{\omega}{k}$: Dispersion relation

⟨ 1st order form of Wave Eq. ⟩

From the wave eq. $\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$ ①

$$E(z,t) = E_0(z,t)\, e^{i(kz - \omega t + \phi)},$$

where E_0 and ϕ are slowly varying.

(i) ∇E : $\frac{\partial E}{\partial z} = \left[\frac{\partial E_0}{\partial z} + E_0\left(ik + i\frac{\partial \phi}{\partial z}\right)\right] e^{i(kz-\omega t+\phi)}$

$\nabla^2 E$: $\frac{\partial^2 E}{\partial z^2} = \left[\cancel{\frac{\partial^2 E_0}{\partial z^2}} + i\frac{\partial E_0}{\partial z}\left(k + \frac{\partial \phi}{\partial z}\right) + iE_0\cancel{\frac{\partial^2 \phi}{\partial z^2}}\right] e^{i(kz-\omega t+\phi)}$

$+ i\left[\frac{\partial E_0}{\partial z} + iE_0\left(k + \frac{\partial \phi}{\partial z}\right)\right]\left(k + \frac{\partial \phi}{\partial z}\right) e^{i(kz-\omega t+\phi)}$

By slowly varying property of E_0 and ϕ,

$\left(\frac{\partial E_0}{\partial z} \ll kE_0 \ ; \ \frac{\partial E}{\partial t} \ll \omega E_0 \ ; \ \frac{\partial \phi}{\partial z} \ll k \ ; \ \frac{\partial \phi}{\partial t} \ll \omega\right).$

the 2nd derivatives of E_0 & ϕ go to zero!

$\therefore \nabla^2 E = \left(2ik\frac{\partial E_0}{\partial z} + 2ikE_0\frac{\partial \phi}{\partial z} - k^2 E_0\right) e^{i(kz-\omega t+\phi)}$

(ii) $\frac{\partial E}{\partial t} = \left[\frac{\partial E_0}{\partial t} + iE_0\left(-\omega + \frac{\partial \phi}{\partial t}\right)\right] e^{i(kz-\omega t+\phi)}$

$\frac{\partial^2 E}{\partial t^2} = \left[\frac{\partial^2 E_0}{\partial t^2} + i\frac{\partial E_0}{\partial t}\left(-\omega + \frac{\partial \phi}{\partial t}\right) + iE_0\frac{\partial^2 \phi}{\partial t^2}\right] e^{i(kz-\omega t+\phi)}$

$+ i\left[\frac{\partial E_0}{\partial t} + iE_0\left(-\omega + \frac{\partial \phi}{\partial t}\right)\right]\left(-\omega + \frac{\partial \phi}{\partial t}\right) e^{i(kz-\omega t+\phi)}$

$= \left(-2i\omega\frac{\partial E_0}{\partial t} - 2i\omega E_0\frac{\partial \phi}{\partial t} - \omega^2 E_0\right) e^{i(kz-\omega t+\phi)}$

By inserting (i) & (ii) into equation ①,

$$2ik\frac{\partial E_0}{\partial z} + \frac{1}{c^2}(2i\omega)\frac{\partial E_0}{\partial t} = \left(k^2 - \frac{\omega^2}{c^2}\right)E_0 = 0$$

$$2ik\frac{\partial \phi}{\partial z} + \frac{1}{c^2}(2i\omega)\frac{\partial \phi}{\partial t} = 0$$

$$\therefore \frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t} = 0 \ ; \ \frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0$$

(Slowly Varying Amplitude and Phase)

< Speed of light in a medium >

$$\mu_0 \to \mu \quad ; \quad \epsilon_0 \to \epsilon \quad ; \quad c \to u$$

$$u = \frac{1}{\sqrt{\epsilon\mu}} \quad ; \quad K = \frac{\epsilon}{\epsilon_0} \quad ; \quad K_m = \frac{\mu}{\mu_0}$$

$$= \frac{1}{\sqrt{KK_m}} \frac{1}{c} \quad \equiv \frac{n}{c}$$

$\therefore$ Refractive index $n = \sqrt{KK_m}$

In a nonmagnetic medium ($K_m = 1$),

$$n = \sqrt{K}$$

Truth : n is a function of λ. $\to$ Dispersion

< Waves in 1-D >

$$\nabla^2 u = \frac{1}{u^2} \frac{\partial^2 u}{\partial t^2} ,$$

$$u(z,t) = u_0 \cos(kz - \omega t) \quad : \text{General sol.}$$

$$\to \quad u = \frac{\omega}{k} \qquad \hookrightarrow u_0 e^{i(kz-\omega t)}$$

pf).
$$\phi = kz - \omega t$$
$$\Delta\phi = k\Delta z - \omega\Delta t \quad \text{for a monochromatic plane harmonic wave.}$$
$$= 0$$

$$\therefore \frac{\Delta z}{\Delta t} = \frac{\omega}{k} \Rightarrow u \quad : \text{phase velocity}$$

- period, T : $\lambda = uT$
- frequency ν : $\nu = \frac{u}{\lambda} = \frac{1}{T}$
- Coherence vs. Incoherence

1.5 Group velocity

· The harmonic wave is not monochromatic

$$\begin{cases} \omega \to \omega \pm \Delta\omega \\ k \to k \pm \Delta k \end{cases}$$

$$U = U_0 \left\{ e^{i[(k+\Delta k)z - (\omega+\Delta\omega)t]} + e^{i[(k-\Delta k)z - (\omega-\Delta\omega)t]} \right\}$$

$$= U_0 e^{i(kz-\omega t)} \left\{ e^{i(\Delta kz - \Delta\omega t)} + e^{-i(\Delta kz - \Delta\omega t)} \right\}$$

$$= 2U_0 e^{i(kz-\omega t)} \underbrace{\cos(z\Delta k - t\Delta\omega)}_{\text{modulation term}}$$

(See Fig. 1.3)

Group velocity u_g:

$$u_g = \frac{\Delta\omega}{\Delta k} \to \frac{d\omega}{dk}$$

· In a dispersive medium whose index of refraction is $n = \frac{c}{u}$,

$$\omega = ku = \frac{kc}{n}.$$

· the group velocity u_g,

$$u_g = \frac{d\omega}{dk} = \frac{d}{dk}\left(\frac{kc}{n}\right) = \frac{c}{n} - \frac{ck}{n^2}\frac{dn}{dk}$$

$$= u\left(1 - \frac{k}{n}\frac{dn}{dk}\right).$$

OR $dk = -\frac{2\pi}{\lambda^2}d\lambda \quad \left(k = \frac{2\pi}{\lambda}\right)$

$$u_g = -\frac{\lambda^2}{2\pi}\frac{d}{d\lambda}\left(\frac{2\pi c}{\lambda n}\right) = -\frac{\lambda^2}{2\pi}\left(-\frac{2\pi}{\lambda^2}\frac{c}{n} - \frac{2\pi c}{n^2}\frac{1}{\lambda}\frac{dn}{d\lambda}\right)$$

$$= u\left(1 + \frac{\lambda}{n}\frac{dn}{d\lambda}\right).$$

· For normal dispersion,

$$\frac{dn}{d\lambda} < 0 \rightarrow \therefore u_g < u.$$ ☆