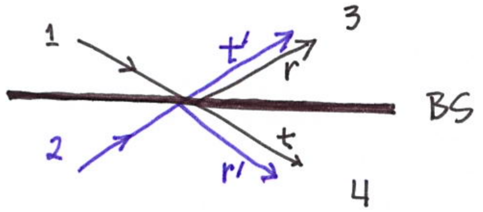
EE5107 April 4, 2018

<Beam Splitter>



: primed notation is for $2 \to 1$.

$$\begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} B \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$$: Matrix formation for linear transformation

$$B = \begin{pmatrix} r & t' \\ t & r' \end{pmatrix}$$

For energy conservation (~~propa~~ probability conservation),

B must be unitary. $\rightarrow$ $B^{+} = B^{-1}$; $B^{+} = \tilde{B}^{*}$

(i) $B^{-1} = \frac{1}{D}\begin{pmatrix} r' & -t' \\ -t & r \end{pmatrix}$; $D = rr' - tt'$

Check : $BB^{-1} = \mathbb{1} \rightarrow \frac{1}{D}\begin{pmatrix} r & t' \\ t & r' \end{pmatrix}\begin{pmatrix} r' & -t' \\ -t & r \end{pmatrix} = \frac{1}{rr'-tt'}\begin{pmatrix} rr'-tt' & rt'-rt' \\ tr'-tr' & r'r-tt' \end{pmatrix}$

$= \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. QED.

(ii) $B^{+} = \begin{pmatrix} r^{*} & t^{*} \\ t'^{*} & r'^{*} \end{pmatrix} = \frac{1}{rr'-tt'}\begin{pmatrix} r' & -t' \\ -t & r \end{pmatrix}$,

Here D does not affect the matrix notation.

$\therefore$ $r^{*} = r'$; $t^{*} = -t'$

For generality r & t are complex.

$\rightarrow$ $r \rightarrow re^{i\phi}$; $t \rightarrow te^{i\theta}$

Then, $re^{-i\phi_r} = re^{i\phi_r}$; $te^{-i\theta_t} = -te^{i\theta_t}$

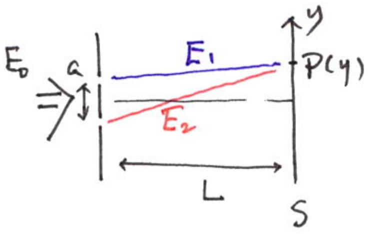
$\cancel{\cos\phi} - i\sin\phi = \cancel{\cos\phi} + i\sin\phi$ $\qquad$ $\cos\theta - \cancel{i\sin\theta} = -\cos\theta - \cancel{i\sin\theta}$

$\phi_r = \pi$. $\qquad$ $\theta_t = \frac{\pi}{2}$

$\therefore$ $\phi_r - \phi_t = \frac{\pi}{2}$; For 50% BS, ($|r| = |t|$) $B = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}$

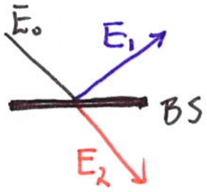
<Young's double slit exp. vs. BS>

April 4, 2018



$E_0 \longrightarrow E_1 + E_2$; E_1 & E_2 are coherent!

<Young's Model>



$E_0 \rightarrow E_1 + E_2$;

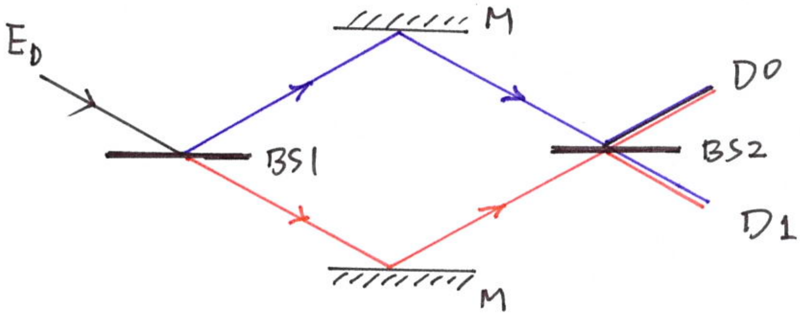
E_1 & E_2 are coherent!

(With $\frac{\pi}{2}$ phase shift)

<BS Model>

< Interference formation in the BS model >

· Mach-Zehnder interferometer



Here, the phase shift on the mirror (M) is universal (global),
so that there is no relative phase change on the outputs (D0 & D1)

Let the input matrix be $E_0 \begin{pmatrix} 1 \\ 0 \end{pmatrix}$.

Then the output is $\begin{pmatrix} D_0 \\ D_1 \end{pmatrix}$, where $\begin{pmatrix} D_0 \\ D_1 \end{pmatrix} = (BS2)(BS1)\begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$$\rightarrow \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 & i \\ i & 1 \end{pmatrix}\begin{pmatrix} 1 \\ i \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 \\ 2i \end{pmatrix} = i\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Thus, the output is on only D1 detector!