Hw#2

Prove that cylindrical waves do not satisfy the wave equation. See the lecture note on week 2-2.

→ In a cylindrical coord.,

$$\psi = \frac{A}{\sqrt{\rho}} e^{i(k\rho \pm \omega t)}$$

(i) $\nabla = \frac{\partial}{\partial \rho}\hat{\rho} + \cancel{\frac{1}{\rho}\frac{\partial}{\partial \phi}\hat{\phi}} + \cancel{\frac{\partial}{\partial z}\hat{k}}$

(ii) $\nabla^2\psi = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial \psi}{\partial \rho}\right) = \frac{1}{\rho}\frac{\partial \psi}{\partial \rho} + \frac{\partial^2 \psi}{\partial \rho^2}$

A. $\frac{\partial \psi}{\partial \rho} = \left(ik - \frac{1}{2\rho}\right)\psi = ik\frac{A}{\sqrt{\rho}}e^{i(k\rho \pm \omega t)} - \frac{A}{2\rho^{3/2}}e^{i(k\rho \pm \omega t)}$

B. $\frac{\partial^2 \psi}{\partial \rho^2} = \frac{\partial}{\partial \rho}\left(\frac{\partial \psi}{\partial \rho}\right) = ik\left(ik - \frac{1}{2\rho}\right)\psi + \frac{3}{4}\frac{A}{\rho^{5/2}}e^{i(k\rho \pm \omega t)} - (ik)\frac{A}{2\rho^{3/2}}e^{i(k\rho \pm \omega t)}$

$$= \left(-k^2 - \frac{ik}{\rho} + \frac{3}{4\rho^2}\right)\psi$$

$$\therefore A \times \frac{1}{\rho} + B = \left(-k^2 + \frac{1}{4\rho^2}\right)\psi = \nabla^2\psi \quad ①$$

From the wave equation of (4-2),

$$\frac{\partial^2 \psi}{\partial \rho^2} = \frac{1}{v^2}\frac{\partial^2 \psi}{\partial t^2}.$$

$$\rightarrow \frac{\partial^2 \psi}{\partial t^2} = -\omega^2\psi \quad \therefore \frac{1}{v^2}(-\omega^2)\psi = -k^2\psi \quad ②$$

∴ ① ≠ ②, unless $\rho \rightarrow \infty$!