1.2 Electrical const. & Speed of light

· Maxwell's equations in vacuum :

$$\begin{cases} \nabla \times \bar{E} = -\mu_0 \frac{\partial \bar{H}}{\partial t} \\ \nabla \times \bar{H} = \epsilon_0 \frac{\partial \bar{E}}{\partial t} \end{cases} \qquad \begin{cases} \nabla \cdot \bar{E} = 0 \\ \nabla \cdot \bar{H} = 0 \end{cases}$$

- μ_0 : magnetic permeability ($= 4\pi \times 10^{-7}$ H/m)
- ϵ_0 : electric permittivity ($= 8.85 \times 10^{-12}$ F/m)

(Wave equation)

(i) $\nabla \times (\nabla \times E) = \nabla(\nabla \cdot E) - \nabla^2 E = -\nabla^2 E$

$$\Rightarrow -\mu_0 \frac{\partial}{\partial t}(\nabla \times \bar{H}) = -\epsilon_0 \mu_0 \frac{\partial^2 E}{\partial t^2}$$

(ii) $\nabla \times (\nabla \times \bar{H}) = \nabla(\nabla \cdot H) - \nabla^2 H = -\nabla^2 H$

$$\Rightarrow \epsilon_0 \frac{\partial}{\partial t}(\nabla \times \bar{E}) = -\epsilon_0 \mu_0 \frac{\partial^2 H}{\partial t^2}$$

$$\rightarrow \nabla^2(\ ) = \frac{1}{c^2} \frac{\partial^2}{\partial t^2}(\ ),$$

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} \sim 3 \cdot 10^8 \ (m/s)$$

Q. What is the form of ( ) ?

· Traveling wave

$$f(z,t) \rightarrow f(z - vt, 0)$$

General solution : $f = f_0 e^{i(kz - \omega t + \phi)}$

$\rightarrow c = \frac{\omega}{k}$: Dispersion relation

⟨ 1st order form of Wave Eq. ⟩

From the wave eq. $\nabla^2 E - \frac{1}{c^2}\frac{\partial^2 E}{\partial t^2} = 0$ ①

$$E(z,t) = E_0(z,t)\, e^{i(kz-\omega t+\phi)},$$

where E_0 and ϕ are slowly varying.

(i) ∇E : $\frac{\partial E}{\partial z} = \left[\frac{\partial E_0}{\partial z} + E_0\left(ik + i\frac{\partial \phi}{\partial z}\right)\right] e^{i(kz-\omega t+\phi)}$

$\nabla^2 E$: $\frac{\partial^2 E}{\partial z^2} = \left[\cancel{\frac{\partial^2 E_0}{\partial z^2}}^{0} + i\frac{\partial E_0}{\partial z}\left(k + \cancel{\frac{\partial \phi}{\partial z}}^{0}\right) + iE_0\cancel{\frac{\partial^2 \phi}{\partial z^2}}^{0}\right] e^{i(kz-\omega t+\phi)}$

$+ i\left[\frac{\partial E_0}{\partial z} + iE_0\left(k + \frac{\partial \phi}{\partial z}\right)\right]\left(k + \frac{\partial \phi}{\partial z}\right) e^{i(kz-\omega t+\phi)}$

By slowly varying property of E_0 and ϕ,

$\left(\frac{\partial E_0}{\partial z} \ll kE_0 \;;\; \frac{\partial E}{\partial t} \ll \omega E_0 \;;\; \frac{\partial \phi}{\partial z} \ll k \;;\; \frac{\partial \phi}{\partial t} \ll \omega\right)$.

the 2nd derivatives of E_0 & ϕ go to zero!

$\therefore \nabla^2 E = \left(2ik\frac{\partial E_0}{\partial z} + 2ikE_0\frac{\partial \phi}{\partial z} - k^2 E_0\right) e^{i(kz-\omega t+\phi)}$

(ii) $\frac{\partial E}{\partial t} = \left[\frac{\partial E_0}{\partial t} + iE_0\left(-\omega + \frac{\partial \phi}{\partial t}\right)\right] e^{i(kz-\omega t+\phi)}$

$\frac{\partial^2 E}{\partial t^2} = \left[\frac{\partial^2 E_0}{\partial t^2} + i\frac{\partial E_0}{\partial t}\left(-\omega + \frac{\partial \phi}{\partial t}\right) + iE_0\frac{\partial^2 \phi}{\partial t^2}\right] e^{i(kz-\omega t+\phi)}$

$+ i\left[\frac{\partial E_0}{\partial t} + iE_0\left(-\omega + \frac{\partial \phi}{\partial t}\right)\right]\left(-\omega + \frac{\partial \phi}{\partial t}\right) e^{i(kz-\omega t+\phi)}$

$= \left(-2i\omega\frac{\partial E_0}{\partial t} - 2i\omega E_0\frac{\partial \phi}{\partial t} - \omega^2 E_0\right) e^{i(kz-\omega t+\phi)}$

By inserting (i) & (ii) into equation ①,

$2ik\frac{\partial E_0}{\partial z} + \frac{1}{c^2}(2i\omega)\frac{\partial E_0}{\partial t} = \left(k^2 - \frac{\omega^2}{c^2}\right)E_0 = 0$

$2ik\frac{\partial \phi}{\partial z} + \frac{1}{c^2}(2i\omega)\frac{\partial \phi}{\partial t} = 0$

$\therefore \frac{\partial E_0}{\partial z} + \frac{1}{c}\frac{\partial E_0}{\partial t} = 0 \;;\; \frac{\partial \phi}{\partial z} + \frac{1}{c}\frac{\partial \phi}{\partial t} = 0$

(Slowly Varying Amplitude and Phase)