EC5103 April 26, 2017

Quiz#4

Name:

Determine the Fourier series ofr the function of spatial period L given by

$$f(x) = \begin{cases} -1, & \frac{-L}{2} < x < 0 \\ +1, & 0 < x < \frac{+L}{2} \end{cases}$$

Sol) Odd function. $a_n = a_0 = 0$

$$f(x) = \sum_{m=1}^{\infty} b_m \sin mkx \quad , \quad b_m = \frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} f(x) \sin mkx \, dx$$

$$\rightarrow b_m = \frac{2}{L}\left\{ \int_{-\frac{L}{2}}^{0} -\sin mkx \, dx + \int_{0}^{\frac{L}{2}} \sin mkx \, dx \right\}$$

$$= \frac{2}{L \cdot mk}\left\{ \cos mkx \Big|_{-\frac{L}{2}}^{0} - \cos mkx \Big|_{0}^{\frac{L}{2}} \right\}$$

$$= \frac{2}{mkL}\left\{ 1 - \cos\frac{mkL}{2} - \cos\frac{mkL}{2} + 1 \right\} = \frac{4}{mkL}\left[1 - \cos\frac{mkL}{2} \right]$$

$$k = \frac{2\pi}{L} \qquad \therefore \underline{b_m = \frac{2}{m\pi}(1 - \cos m\pi)} \quad , \quad m = 1, 2, 3, \dots$$

$$\therefore f(x) = \frac{2}{\pi}\sum_{m=1}^{\infty}\left(\frac{1-\cos m\pi}{m}\right)\sin mkx$$

$$= \frac{2}{\pi}\left(2\sin kx + \frac{2}{3}\sin 3kx + \frac{2}{5}\sin 5kx + \cdots \right)$$

$$= \frac{4}{\pi}\left(\sin kx + \frac{1}{3}\sin 3kx + \frac{1}{5}\sin 5kx + \cdots \right)$$

Check!

(i) $a_0 = \frac{2}{L}\int_{-\frac{L}{2}}^{\frac{L}{2}} f(x)\,dx = \frac{2}{L}\left\{ \int_{-\frac{L}{2}}^{0}(-1)\,dx + \int_{0}^{\frac{L}{2}}(+1)\,dx \right\} = \frac{2}{L}\left\{ (-1)\left(0 - \frac{-L}{2}\right) + (+1)\left(\frac{L}{2} - 0\right) \right\} = 0$

(ii) $a_m = \frac{2}{L}\left\{ \int_{-\frac{L}{2}}^{0} -\cos mkx \, dx + \int_{0}^{\frac{L}{2}} \cos mkx \, dx \right\}$

$= \frac{2}{L}\left\{ \frac{-1}{mk}\sin mkx \Big|_{-\frac{L}{2}}^{0} + \frac{1}{mk}\sin mkx \Big|_{0}^{\frac{L}{2}} \right\} = \frac{2}{mkL}\left\{ +\sin\left(-\frac{mkL}{2}\right) + \sin\frac{mkL}{2} \right\} =$