B. Super-sensitivity

· Classical limit : Shot-noise limit

$$\Delta\varphi = \frac{1}{\sqrt{N}} \qquad \left(\Delta P(\varphi) \Big/ \frac{\partial P(\varphi)}{\partial\varphi} \right)$$

probability (Intensity)

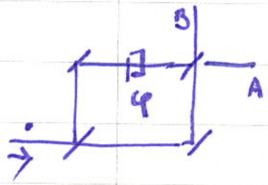
· Quantum limit : Heisenberg limit

$$\Delta\varphi = \frac{1}{N}$$

· $\Delta P(\varphi) = \langle \hat{P}^2 \rangle - \langle \hat{P} \rangle^2 = P(\phi)\,(1 - P(\phi))$

· For $N=1$,

$$P(\phi) = \cos^2\phi$$



$$\begin{cases} I_A = I_0 \cos^2 \frac{\varphi}{2} \\ I_B = I_0 \sin^2 \frac{\varphi}{2} \end{cases}$$

· statistical average $\sum_{j=1}^{N} x_j / N$, $x_j \in \{0, 1\}$

· How to measure φ ?

− By monitoring the MZI output intensities.

· If $I_A - I_B = I_0$, $\varphi = 0$

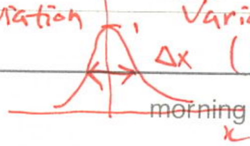
· If $I_A - I_B = -I_0$, $\varphi = \pi$

· How precisely measure φ ?

$\rightarrow \Delta\varphi = 1/\sqrt{N}$, N : # of measurements

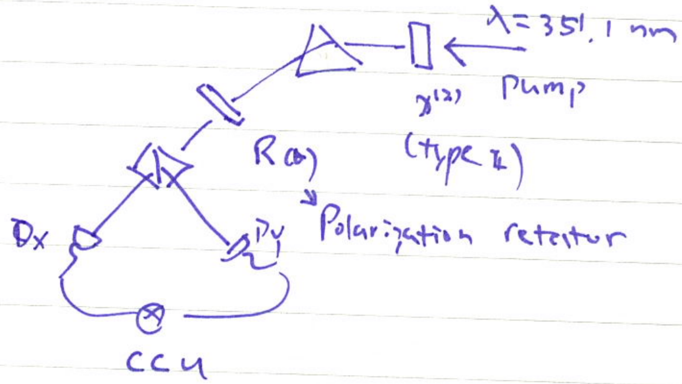
· Due to the uncorrelated measurements x_j,

the error of $\cos^2\frac{\varphi}{2}$ is $\underbrace{\Delta\left(\sum_j^N x_j/N\right)}_{\text{Standard Deviation}} = \sqrt{\underbrace{\sum_j^N \Delta^2 x_j}_{\text{Variance}}/N} = \overset{\text{SNL}}{\frac{\Delta x}{\sqrt{N}}}$.



Supersensitivity

ex. 1) Quantum Semiclass. Opt. 10, 493 ('98)



For M coincidence counts,

$$\langle M\rangle = \langle N\rangle \alpha_x\alpha_y \cos^2 2\theta$$

$$\langle M^2\rangle = \langle N\rangle \alpha_x\alpha_y \cos^2 2\theta\,(1-\alpha_x\alpha_y\cos^2 2\theta) + \langle N^2\rangle(\alpha_x\alpha_y\cos^2 2\theta)^2$$

With perfect detectors ($\alpha_x=\alpha_y=1$), and photon numer discrimation ~~capability~~ capability, $\Delta N = 0$.

$$\rightarrow \langle(\Delta M)^2\rangle = \langle M^2\rangle - \langle M\rangle^2 = \frac{1}{4}\langle N\rangle \sin^2(4\theta).$$

If photon number cannot be discrimated,

$$\langle(\Delta M)^2\rangle = \langle N\rangle\alpha_x\alpha_y\cos^2 2\theta,$$ where $\langle(\Delta N)^2\rangle \sim \langle N\rangle$ due to Poisson.

Thus, $$\frac{\langle(\Delta M)^2\rangle}{\langle(\Delta\theta)^2\rangle} \sim \left(\frac{d\langle M\rangle}{d\theta}\right)^2$$

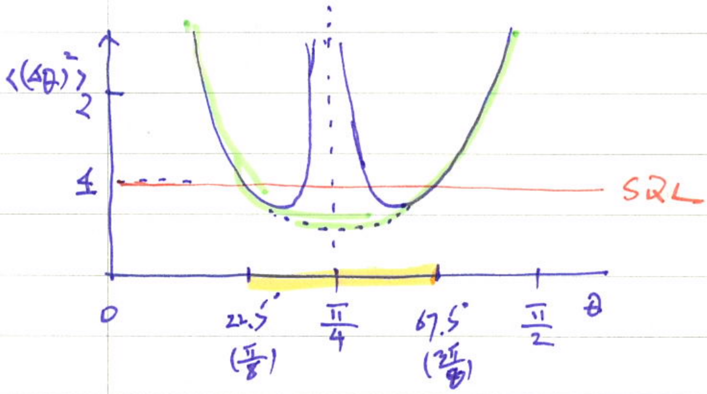
$$\rightarrow \langle(\Delta\theta)^2\rangle = \frac{1}{16\langle N\rangle}, \quad \frac{d(\cos^2 2\theta)}{d\theta} = 4\cos 2\theta\sin 2\theta = 2\sin 4\theta.$$

But, $$\langle(\Delta\theta)^2\rangle_{SQL} = \frac{1}{8(\alpha_x\alpha_y\langle N\rangle)}$$ } ⇒ twice enhanced!

* $|\psi\rangle = -\frac{1}{\sqrt{2}}\sin 2\theta\,|2\rangle_x|0\rangle_y + \cos 2\theta\,|1\rangle_x|1\rangle_y + \frac{1}{\sqrt{2}}\sin 2\theta\,|0\rangle_x|2\rangle_y$

For classical detection, $|\psi\rangle = \frac{1}{\sqrt{2}}\sin 2\theta$. $\rightarrow \langle M\rangle = \frac{1}{2}\sin^2 2\theta$

- $\langle M^2\rangle = \langle N\rangle \sin^2 2\theta\,(1-\sin^2 2\theta) - \langle N^2\rangle \sin^4 2\theta$
- $\langle(\Delta M)^2\rangle = \langle M^2\rangle - \langle M\rangle^2 = \frac{1}{8}\langle N\rangle\sin^2(4\theta)$.
- $\left(\frac{d\langle M\rangle}{d\theta}\right)^2 = (2\sin 2\theta\cos 2\theta)^2 = \sin^2(4\theta)$ $\quad\therefore \langle(\Delta\theta)^2\rangle = \frac{\langle(\Delta M)^2\rangle}{\left(\frac{d\langle M\rangle}{d\theta}\right)^2} = \frac{1}{8\langle N\rangle}$.

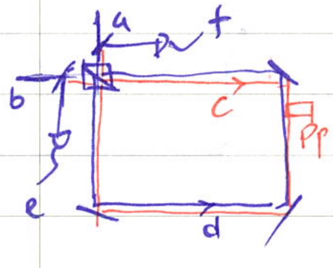


$$\frac{\langle(\Delta\theta)^2\rangle}{\langle(\Delta\theta)^2\rangle_{SQL}} = \left(\frac{1+\eta}{2\eta^2}\right)\frac{(1+\eta\cos 4\theta)}{\sin^2(4\theta)}.$$

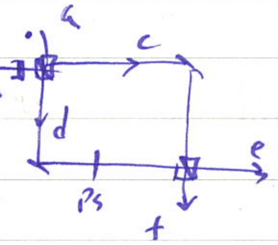
- Super-sensitivity : $\frac{\pi}{8} \leq \theta \leq \frac{3\pi}{8}$, except $\theta \in \{40, 50°\}$

< Super-sensitivity >

Source: BBO (type I)   참고 : Science 316, 726 ('07)



⟺



- $P_{ef} = (1-\cos 2\phi)/2$
- Input state : $|22\rangle_{ab}$
- Inside Sagnac :

$$|\psi\rangle = \sqrt{\tfrac{3}{8}}\left(|40\rangle_{cd} + |04\rangle_{cd}\right) + \tfrac{1}{2}|22\rangle_{cd}$$

- Outside : $\underline{|31\rangle_{ef}} + |13\rangle_{ef}$

$\rightarrow P_{3ef} = \frac{3}{8}(1-\cos 4\phi)/2$: phase super resolution

- Due to $|22\rangle_{cd}$, η efficiency is inserted for $|\psi\rangle$.

$\rightarrow P = \eta_i (1-\cos N\phi)/2$

- To beat SQL, N-photon interference visibility

$\checkmark > 1/\sqrt{\eta N}$ $\left(= 1/\sqrt{\tfrac{3}{8}\cdot 4} = \sqrt{2/3} = 81.6\,\% \right)$

→ Super sensitivity

- $P_e = (1-\cos\phi)/2$

↓ repeat N times

$\Delta\phi = 1/\sqrt{N}$: SQL

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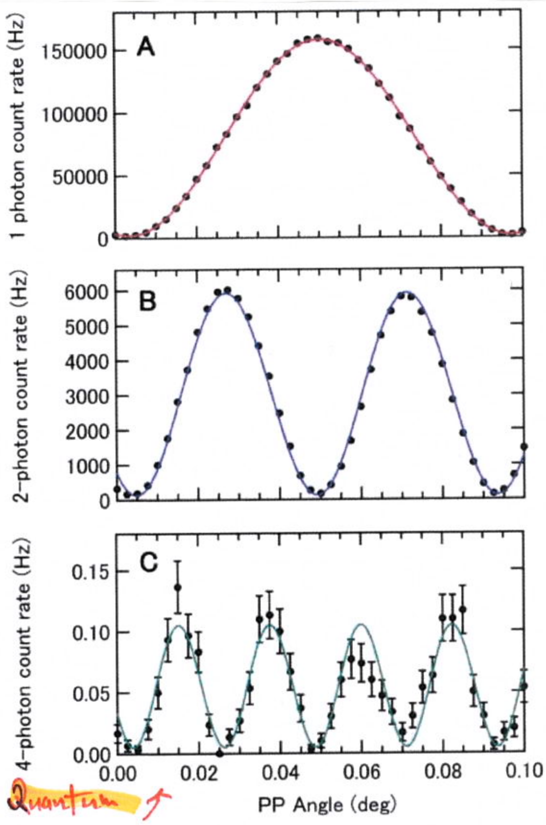
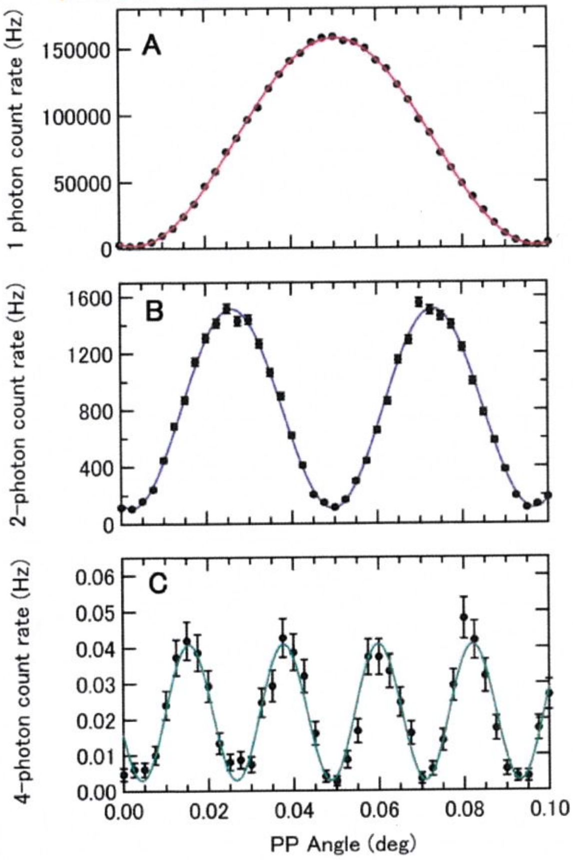
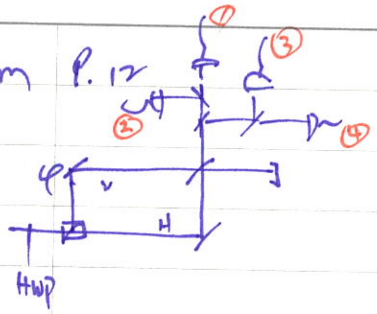


Quantum ↑

classical ↘



From P. 17



- Perfect visibility due to single-input MZI ✗
- coincidence-detection-based post-measurement resulting perfect photon number for four-coincidence / two / one cases.

- Thus,

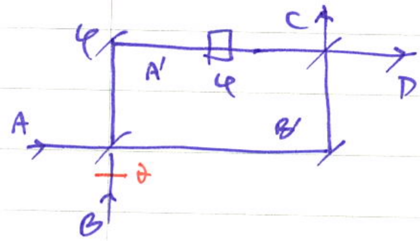
$$V > \frac{1}{\sqrt{N}} \quad \text{to beat SQL.}$$

i) For single-photon case, no beating occurs.

ii) For two-photon case, $V > \frac{1}{\sqrt{2}} \rightarrow$ beat

iii) For four-photon case, $V > \frac{1}{2} \rightarrow$ beat

출처: Science 306, 1330 ('04)



∗ Classical approach for a fixed θ:

$$\begin{pmatrix} C \\ D \end{pmatrix} = (BS)(\varphi)(BS)\begin{pmatrix} A \\ B \end{pmatrix}$$

For $B = Ae^{i\theta}$,

$$\begin{pmatrix} C \\ D \end{pmatrix} = \frac{E_0}{2}\begin{pmatrix} 1-e^{i\varphi} & i(1+e^{i\varphi}) \\ i(1+e^{i\varphi}) & -(1-e^{i\varphi}) \end{pmatrix}\begin{pmatrix} 1 \\ e^{i\theta} \end{pmatrix}$$

$$\rightarrow E_C = \frac{E_0}{2}\left[(1-e^{i\varphi}) + ie^{i\theta}(1+e^{i\varphi})\right]$$

For $\theta = \pm\frac{\pi}{2}$, $E_C = E_0 \begin{cases} -e^{i\varphi} & (+\frac{\pi}{2}) \\ 1 & (-\frac{\pi}{2}) \end{cases}$

$\rightarrow \underline{I_C = I_0}$. (No fringe)

$$\cdot\ P_D = \cos^2\frac{\varphi}{2} \qquad (1+\cos\varphi)$$

$$P_C = \sin^2\frac{\varphi}{2} \qquad (1-\cos\varphi)$$

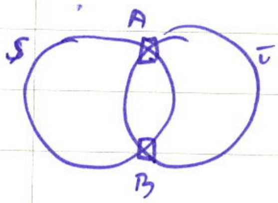
∗∗ For superposed case of $\theta_s \in \{\pm\frac{\pi}{2}\}$,

$$E_C = \frac{E_0}{\sqrt{2}}(1-e^{i\varphi})$$

$$I_C = \frac{I_0}{2}(1-e^{i\varphi})(1-e^{-i\varphi}) = \frac{I_0}{2}(1+1-2\cos\varphi)$$

$$= I_0(1-\cos\varphi)$$

Entangled photon pair from SPDC type II:



$$|\psi\rangle_A = (|s\rangle_A|i\rangle_B + |s\rangle_B|i\rangle_A)/\sqrt{2}$$

$$= (|1\rangle_s|0\rangle_i + |0\rangle_s|1\rangle_i)/\sqrt{2}$$

$$|\psi\rangle_B = (|1\rangle_i|0\rangle_s + |0\rangle_i|1\rangle_s)/\sqrt{2}$$

$$|\psi\rangle_E = \frac{1}{\sqrt{2}}(|\psi\rangle_A|\psi\rangle_B + |\psi\rangle_B|\psi\rangle_A)$$

$$= \frac{1}{\sqrt{2}}(|1\rangle_s|0\rangle_i + |0\rangle_s|1\rangle_i).$$

$$\Downarrow$$

∗ But, no phase relation between the signal and idler photons