CH. 9 Amplification of light : Laser

9.2 Stimulated emission and Thermal radiation

In 1917, Einstein firstly introduced the concept of induced emission of radiation.

Consider a quantized atomic system.

E_2, N_2 ——— $|2\rangle$

E_1, N_1 ——— $|1\rangle$

- E_n : Energy level
- N_n : population (# of atoms/V)

Ex.) $E_n = -\frac{13.6}{n^2}$ (eV) in H.

- Boltzmann distribution

$$P_i = \frac{e^{-E_i/kT}}{\sum_j e^{-E_j/kT}}$$

$(P_i = N_i/N)$ ↖ total #

, P_i : probability of state i

E_j : Energy of state j

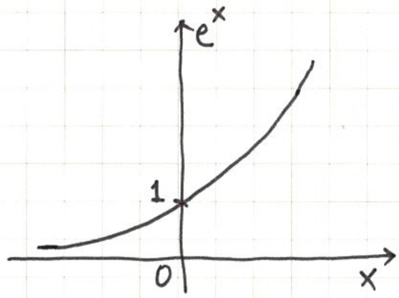
k : Boltzmann const. (1.38×10^{-23})

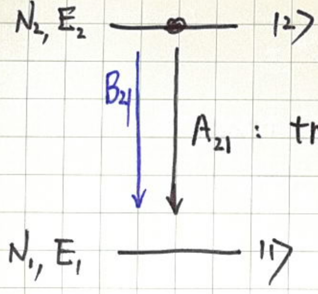
T : temperature (K)

$$\therefore \frac{P_i}{P_j} = e^{+(E_j - E_i)/kT} = \frac{N_i}{N_j}$$

$$\rightarrow \frac{N_2}{N_1} = e^{(E_1 - E_2)/kT}$$

; $E_1 < E_2 \rightarrow N_2 < N_1$





A_{21} : transition probability per unit time from $|2\rangle$ to $|1\rangle$

→ Spontaneous emission

→ Rate

#: $N_2 A_{21}$ (Spontaneous decay number/s)

- B_{21} : transition probability per unit time from $|2\rangle$ to $|1\rangle$ by stimulated emission

  → $B_{21} = B_{12}$

  u_ν : energy density of freq ν :

  #: $N_2 B_{21} u_\nu$ (stimulated decay number/s)

  → Rate

- B_{12} : (stimulated) absorption induced transition probability per unit time from state $|1\rangle$ to $|2\rangle$.

  #: $N_1 B_{12} u_\nu$

- A & B : Einstein A/B coefficients.

- Under thermal equilibrium, both rate must be equal.

$$\rightarrow \quad N_2 A_{21} + N_2 B_{21} u_\nu = N_1 B_{12} u_\nu$$

$$\frac{1}{e^{(E_2-E_1)/kT}-1}$$

$E_2 - E_1 = h\nu$

$$① \quad \therefore u_\nu = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21}}{B_{21}} \frac{1}{\left(\frac{N_1}{N_2}\right) - 1}$$

- From Planck's blackbody radiation, the energy density u_ν is

$$② \quad u_\nu = \frac{8\pi h\nu^3}{c^3} \frac{1}{e^{h\nu/kT}-1}$$

h: Planck const (6.63×10^{-34})

$$\therefore \frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3} \quad ; ① = ②$$

$$\text{OR} \quad \frac{B_{21} u_\nu}{A_{21}} = \frac{1}{e^{h\nu/kT}-1} \quad \left(= \frac{\text{stimulated emission}}{\text{Spontaneous emission}} \right)$$

→ photon numbers per mode !

· From $\frac{1}{e^{h\nu/kT}-1}$, (photon numbers per mode)

At room temperature ($T = 300\,K$),

· $kT = (1.38\times10^{-23})(300) = 4.14\times10^{-21}$

for visible region ($\nu = 6\times10^{14}$ Hz),

· $h\nu = (6.63\times10^{-34})(6\times10^{14}) = 4\times10^{-19}$

$\rightarrow e^{h\nu/kT} \doteqdot 3\times10^{43}$ $\quad\therefore \frac{1}{e^{h\nu/kT}-1} = 0$.

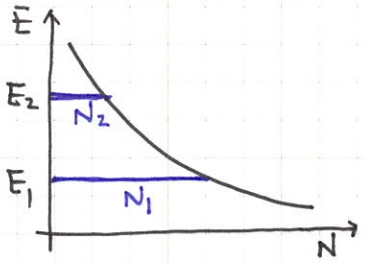
· At room temperature, there is no induced emission but spontaneous emission at thermal equilibrium.

· From $\frac{A_{21}}{B_{21}} = \frac{8\pi h\nu^3}{c^3}$,

A_{21} is dominated for higher freq.

$\rightarrow$ Dificult to get X-ray laser than visible laser!

9.3 Amplification in a medium



<Boltzmann Distribution>

· No stimulated decay if $N_2 < N_1$.

☆ Lasing requirement : $N_2 > N_1$

$\rightarrow$ population inversion