EC4301, 전컴 특론1

2023.11.20

Classical visibility: Intensity-product lower bound

[Analysis]

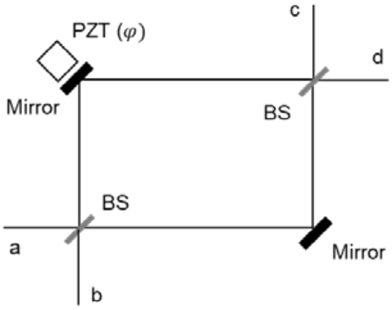

Fig. 1. Schematic of the second-order intensity product. Input photons 'a' and 'b' have random phases.

For the random phased input photons 'a' and 'b,'

$$E_a = E_0 e^{i(kx-\omega t)} e^{i\zeta_a}, \tag{1}$$

$$E_b = E_0 e^{i(kx-\omega t)} e^{i\zeta_b}, \tag{2}$$

where ζ_a and ζ_b are arbitrary to satisfy incoherent condition between them. Using the matrix representation of the beam splitter (BS), the output fields are as follows:

$$\begin{bmatrix} E_c \\ E_d \end{bmatrix} = \frac{E_0}{2} e^{i(kx-\omega t)} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & e^{i\varphi} \end{bmatrix} \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix} \begin{bmatrix} e^{i\zeta_a} \\ e^{i\zeta_b} \end{bmatrix}$$

$$\begin{bmatrix} E_c \\ E_d \end{bmatrix} = \frac{E_0}{2} e^{i(kx-\omega t)} \begin{bmatrix} (1-e^{i\varphi}) & i(1+e^{i\varphi}) \\ i(1+e^{i\varphi}) & -(1-e^{i\varphi}) \end{bmatrix} \begin{bmatrix} e^{i\zeta_a} \\ e^{i\zeta_b} \end{bmatrix}. \tag{3}$$

Thus, the output fields are as follows:

$$E_c = \frac{E_0}{2} e^{i(kx-\omega t)} [(1-e^{i\varphi})e^{i\zeta_a} + i(1+e^{i\varphi})e^{i\zeta_b}], \tag{4}$$

$$E_c = \frac{iE_0}{2} e^{i(kx-\omega t)} [(1+e^{i\varphi})e^{i\zeta_a} + i(1-e^{i\varphi})e^{i\zeta_b}]. \tag{5}$$

The corresponding intensities are:

$$I_c = E_c E_c^* = \frac{I_0}{4}[(1-e^{i\varphi})e^{i\zeta_a} + i(1+e^{i\varphi})e^{i\zeta_b}][(1-e^{-i\varphi})e^{-i\zeta_a} - i(1+e^{-i\varphi})e^{-i\zeta_b}]$$

$$= \frac{I_0}{4}\left[(1-e^{i\varphi})(1-e^{-i\varphi}) + (1+e^{i\varphi})(1+e^{-i\varphi}) + i\left((1+e^{i\varphi})(1-e^{-i\varphi})e^{i(\zeta_b-\zeta_a)} - (1-e^{i\varphi})(1+e^{-i\varphi})e^{-i(\zeta_b-\zeta_a)}\right)\right]$$

$$= I_0[1 - sin\varphi cos(\zeta_b - \zeta_a)], \tag{6}$$

$$I_d = E_d E_d^* = \frac{I_0}{4}[(1+e^{i\varphi})e^{i\zeta_a} + i(1-e^{i\varphi})e^{i\zeta_b}][(1+e^{-i\varphi})e^{-i\zeta_a} - i(1-e^{-i\varphi})e^{-i\zeta_b}]$$

$$= \frac{I_0}{4}\left[(1+e^{i\varphi})(1+e^{-i\varphi}) + (1-e^{i\varphi})(1-e^{-i\varphi}) + i\left((1-e^{i\varphi})(1+e^{-i\varphi})e^{i(\zeta_b-\zeta_a)} - (1+e^{i\varphi})(1-e^{-i\varphi})e^{-i(\zeta_b-\zeta_a)}\right)\right]$$

$$= I_0[1 + sin\varphi cos(\zeta_b - \zeta_a)]. \tag{7}$$

Due to $\delta\zeta = \zeta_b - \zeta_a$ is random, $\langle I_c \rangle = \langle I_d \rangle = I_0$. In other words, no interference fringe is resulted in the individual output fields.

The second-order intensity product between the outputs is as follows:

$$R_{cd} = I_c I_d = I_0^2[1 - sin\varphi cos(\zeta_b - \zeta_a)][1 + sin\varphi cos(\zeta_b - \zeta_a)]$$

$$= I_0^2[1 - sin^2\varphi cos^2(\zeta_b - \zeta_a)]. \tag{8}$$

Thus, the mean intensity product is as follows:

$$\langle R_{cd} \rangle = I_0^2\left[1 - \frac{1}{2} sin^2\varphi\right], \tag{9}$$

where $\langle cos^2(\zeta_b - \zeta_a) \rangle = \frac{1}{2}$.

[Numerical calculations]

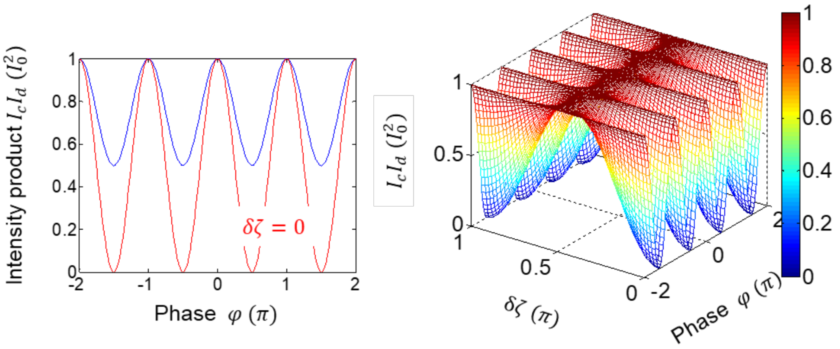

Fig. 2. Numerical calculations of $I_c I_d$. (a) Blue curve: Eq. (9). Red curve: $\delta\zeta = n\pi$. (b) $I_c I_d$ as functions of φ and $\delta\zeta$.