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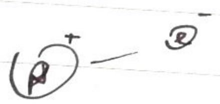
CH. 2 Quantum Measurement and its consequences

1. Classical measurements
    1. unlimited precision measurement
    2. object insensitive (nondemolition) measurement
    3. Predetermined and definite values of measurement.

2. Quantum object (measurement)
    1. Limited precision (standard quantum limit)
    2. object sensitive measurement → projection measurement.
    3. indefinite values of object property → quantum superposition

ex) Bhor's atom model

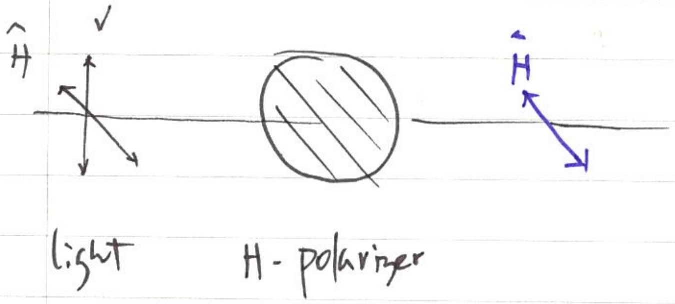


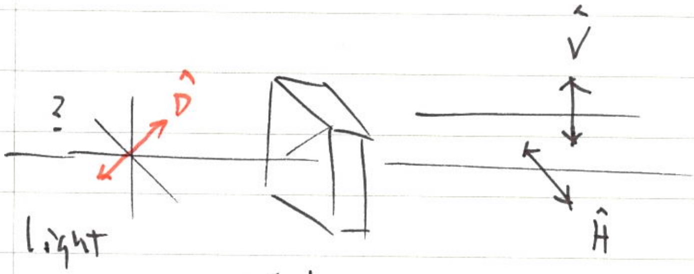
· Classical feature :
1. definite position
2. predetermined property
3. w/ nondemolition measurement

· Quantum feature :
1. indefinite position : probability amplitude
2. $\Delta x \Delta p \geq \hbar$: nondetermined property
3. position or momentum fixed at measurement.

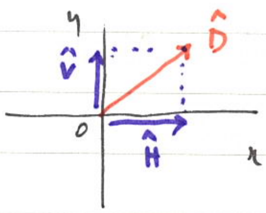
morning glory

· Light polarization

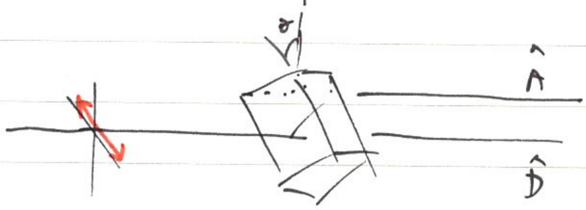




Calcite → Glan-Thompson polarizer
(Thorlabs.com)



Q. What if the input light is $\hat{H}$ polarized ?



Calcite

· physics :

$$\hat{H} = \frac{1}{\sqrt{2}}(\hat{D}+\hat{A})$$

$$\hat{D} = \frac{1}{\sqrt{2}}(\hat{H}+\hat{V})$$

coherent superposition

Q1. What is coherence ?

Q2. What is superposition ?

· Harmonic oscillator

$$\bar{E} = \bar{E}_0 e^{i(kx-\omega t)}$$: Maxwell's Eq.

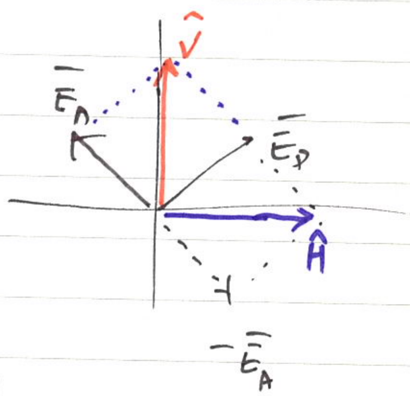
ex) $\bar{E}_D = \hat{D} E_0 e^{i(kx-\omega t)}$

$\bar{E}_A = \hat{A} E_0 e^{i(kx-\omega t+\varphi)}$

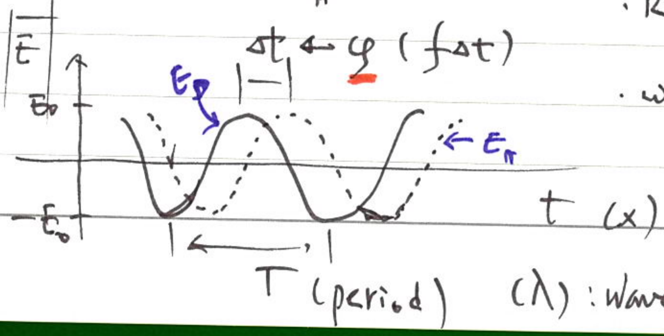
in-phase

if φ is time independent

$\bar{E}_D$ & $\bar{E}_A$ are coherent.



See p.32 (phasor diagram)



(λ) : Wavelength

· $k = \frac{2\pi}{\lambda}$

· $\omega = 2\pi f = \frac{2\pi}{\lambda} c$

$(= kc)$

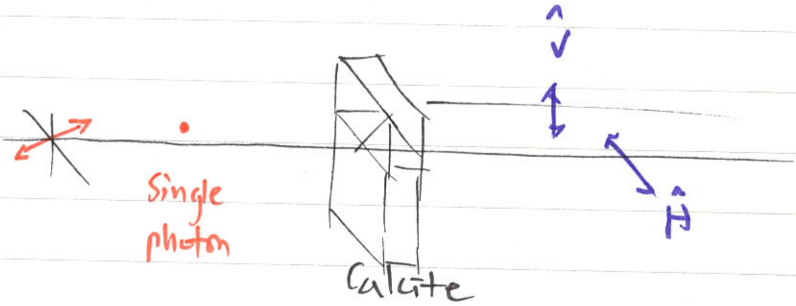
see Fig. 2-8.

Q1. Can we measure the polarization of a single photon?

Q2. What is the physics behind the polarization measurement?

A1. → Any given polarization of a single photon can be decomposed into two perpendicular bases of new polarization axes, in which they are coherent each other.

A2.



$$|\psi\rangle_D = \frac{1}{\sqrt{2}}(\hat{V} + \hat{H})$$

In measurement, either $\hat{H}$ or $\hat{V}$.

→ Randomness. (basis)

Q3. Can you determine the polarization of a single photon by quantum measurement?

A3. For an unknown photon,

No, you cannot. → No cloning thm (1982)

↓

ex) for $\hat{D}$-pol photon,

What is your measurement? → $\hat{H}$ or $\hat{V}$

★ unconditionally secured comm. (QKD).

- What is the difference between the classical and quantum concepts of polarization of light?

→ For an unknown polarization state of a single photon, there is no way to know that polarization by measurements.
→ precision measurement issue

- What if the measured photons are classical light?
  → We can measure the original polarization state!

✓ Discussion: what causes the difference?

- Max Born (1925): Born's rule

- Probability amplitude in measurement. : harmonic oscillator

↑ classical — quantum

$$\bar{A} = |A| e^{i\varphi_1} = |A| (\cos\varphi_1 + i\sin\varphi_1)$$

$$\bar{B} = |B| e^{i\varphi_2} = |B| (\cos\varphi_2 + i\sin\varphi_2)$$

$$|\bar{P}| = \bar{A}\cdot\bar{B} = |A||B| \left(\cos\varphi_1\cos\varphi_2 - \sin\varphi_1\sin\varphi_2 + i(\cos\varphi_1\sin\varphi_2 - \sin\varphi_1\cos\varphi_2) \right)$$

$$= |A||B| \left(\cos(\varphi_1+\varphi_2) + i\sin(\varphi_1-\varphi_2) \right)$$

$$|A|^2 = \bar{A}\cdot\bar{A}^* = |A|^2 (\cos\varphi_1 + i\sin\varphi_1)(\cos\varphi_1 - i\sin\varphi_1)$$

$$= |A|^2 (\cos^2\varphi_1 + \sin^2\varphi_1 + i(\cancel{\sin\varphi_1\cos\varphi_1})) = |A|^2$$

Normalization

- $$\bar{C} = (\bar{A} + \bar{B})\frac{1}{\sqrt{2}} = |A| (e^{i\varphi_1} + e^{i\varphi_2}), \quad \text{for } |\bar{B}| = |\bar{A}|$$

$$\bar{C}\bar{C}^* = |A|^2 (e^{i\varphi_1} + e^{i\varphi_2})(e^{-i\varphi_1} + e^{-i\varphi_2})$$

$$= |A|^2 (1 + 1 + e^{i(\varphi_1-\varphi_2)} + e^{-i(\varphi_1-\varphi_2)})$$

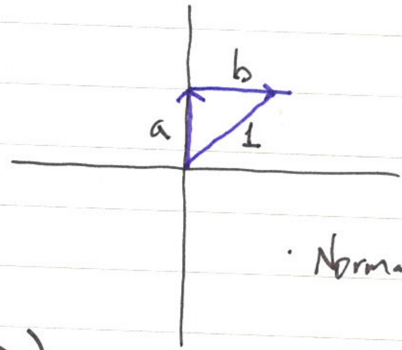
$$= 2|A|^2 = (1 + \underline{\cos\delta\varphi}), \quad \delta\varphi = \varphi_1 - \varphi_2$$

↑ interference.

- Normalization
→ Sum of probability amplitudes : 1.

Discussion 1

For Fig. 2.9,



Born's rule:

$|\psi\rangle = |a\rangle + |b\rangle$

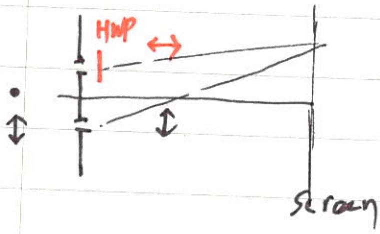
$= \frac{1}{\sqrt{2}} (|V\rangle + |H\rangle)$

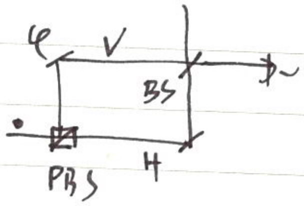
· Normalization : $|a| = |b| = \frac{1}{\sqrt{2}}$.

$\langle\psi|\psi\rangle = \frac{1}{2}\left(\langle V|V\rangle + \langle H|H\rangle + \underbrace{\langle V|H\rangle + \langle H|V\rangle}\right)$

$= 1$, by classical feature as well as by quantum feature due to $\langle V|H\rangle = 0$.

< Experiments >





Q. Interferance fringes on screen?

A. No, due to the orthogonal bases of polarizations.

Q. What if a polarization rotation is conducted in one arm?

A. Quantum eraser.

HW1) Report : Delayed-choice quantum eraser.

due in 4 week! by 10/2

Discussion 2

· photon characteristics : wave - particle duality
(de Broglie)

Q. What is the photon nature?
(Is it a paricle or a wave?)

A. determined by measurements.
→ See the quantum eraser!
(BS Ham, Sci Rep. 13, 9758 ('23))

· <Measurement complementarity>

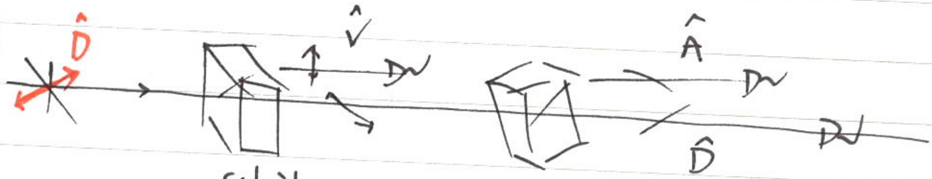
Fact : The measurement outcome depends on your measurement set-up.

1. With H-V polarizer set-up, you can measure either H or V.
2. With D-A polarizer set-up, you can measure either D or A.

However, you cannot use both!

→ complementarity of choosing polarizer set-ups.

Sequential measure?



calcite

Fig. 2.11 Sequential measurements

Outcome : 50% chance ↔ randomness

→ no information

· Bohr's complementarity

→ applies to a conjugate variable set. ← Fourier transform duals

ex) position and momentum

Energy and time ; Doppler and Range

In quantum mechanics, btwn noncommutable variables

$[x, p] = \hat{x}\hat{p} - \hat{p}\hat{x} = i\hbar$ → $\Delta x \Delta p \geq \frac{\hbar}{2}$

→ Heisenberg's uncertainty

Conclusion

- We cannot determine the polarization of an individual (single) photon : quantum realm.
- However, we can determine it if the beam is composed of many photons : classical realm

Quantum superposition (of a single photon) vs. quantum correlation (btwn a pair of photons)

↓

Q. What is the phase of a single photon?

A. indefinite

by $\Delta n \Delta \varphi \geq 1$ · $\underline{\Delta E \Delta t} = hf \Delta n \Delta \varphi \frac{1}{f}$

$= h \underline{\Delta n \Delta \varphi}$

Q. Does simply a large collection of individual photon's (objects) result in the definite classical feature ?

A. Randomness results in no information of it.