EC5103

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Quiz#2

(a) Show that if the maximum positive displacement of a sinusoidal wave occurs at distance x_0 cm from the origin when t=0, its initial phase angle φ_0 is given by:

$$\varphi_0 = \frac{\pi}{2} - \left(\frac{2\pi}{\lambda}\right) x_0,$$

where the wavelength λ is in centimeters.

(b) Determine the initial phase when $\lambda = 10$ cm and $x_0 = 0, \frac{5}{6}, \frac{5}{2}, 5$ and $-\frac{1}{2}$ cm.

(Solution)

(a) $y(x_0) = A\sin(2\pi x_0/\lambda + \varphi_0)$ ← $\sin(kx - \omega t + \varphi_0)$

To be maximally displaced,

$y(x_0) = A \rightarrow 2\pi x_0/\lambda + \varphi_0 = \pi/2$

$\therefore \varphi_0 = \frac{\pi}{2} - \frac{2\pi}{\lambda} x_0$.

(b) $\varphi_0^{(0)} = \frac{\pi}{2} - \frac{2\pi}{10} \cdot 0 = \frac{\pi}{2}$ ← $\varphi_0(x_0) = \frac{\pi}{2} - \frac{2\pi}{\lambda} x_0$

$\varphi_0\left(\frac{5}{6}\right) = \frac{\pi}{2} - \frac{2\pi}{10} \cdot \frac{5}{6} = \frac{\pi}{2} - \frac{\pi}{6} = \frac{\pi}{3}$.

$\varphi_0\left(\frac{5}{2}\right) = \frac{\pi}{2} - \frac{2\pi}{10} \frac{5}{2} = 0$

$\varphi_0(5) = \frac{\pi}{2} - \frac{2\pi}{10} 5 = -\frac{\pi}{2}$.

$\varphi_0\left(-\frac{1}{2}\right) = \frac{\pi}{2} + \frac{2\pi}{10} \frac{1}{2} = \frac{3}{5}\pi$.