⟨ Quantum Sensing ⟩

· Two-mode input state

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\overset{\uparrow V}{|1\rangle_A}\overset{\downarrow H}{|0\rangle_B} + \overset{\downarrow H}{|0\rangle_A}\overset{\uparrow V}{|1\rangle_B}\right)$$: two-photon system

ex) Entangled state : 2 photon

NOON state : N photon

$$|\psi\rangle_{NOON} = \frac{1}{\sqrt{2}}\left(|N\rangle_A|0\rangle_B + |0\rangle_A|N\rangle_B\right)$$

Squeezed-Vacuum state

· Applications

- Gravitation-wave detection : LIGO (MZI, squeezed-light-based)
- Atomic clocks - GPS — atom MZI
- Gravity map - gravimeter (GPS-based)
- Lithography - PBW-based

(· no non-classical light — Atomic clocks, Gravity map)

⟨ Sensing in Heisenberg limit ⟩

- Quantum metrology :
- Quantum Gyroscope
- Quantum radar (lidar)
- Quantum Internet
- Quantum wireless communications
- etc.

Q. why optical phase measurements?

A. For precision measurements of distance, position, displacement, acceleration, velocity (Doppler shift), rotation, gravity, gravitational wave, magnetic, etc.

morning glory

~~Theory:~~

~~1. Sensitivity~~

출처: Review of Modern Physics 89, 035002 (2017)

· Crietria of quantum sensing

1. Use of quantum object to measure a physical quantity — Quantized enenerg structures
2. Use of quantum coherence ...
3. ✓ Use of quantum entanglement

Ex. for category 1. and 2

a. Neutral atoms as magnetic field sensors
— magnetometry

b. Trap Ions — electric & magnetic sensors

c. Rydberg atoms : highy excied electronic states
— electric dipole (d) $\bar{d} = q\bar{l}$
— Stark shift

d. Atomic clock

e. Solid-state spins — NMR, NV center

f. SQUID — electric current, magnetic field

Ex. sensing AC signals

a. Ramsey

b. photon-echo

c. AFC (atomic-frequency comb) — q. memory

d. dynamic decoupling (NMR)

* Main problem : decoherence

· Quantum Sensing for categori 3.

1. Entangled state : GHZ, NOON
   two-mode

   · GHZ : $|\psi\rangle_G = (|00\cdots0\rangle + |11\cdots1\rangle)/\sqrt{2}$

   · NOON : $|\psi\rangle_N = (|0\rangle_a|N\rangle_b + |N\rangle_a|0\rangle_b)/\sqrt{2}$

   In an interferometer (MZI), where path a is applied by $|N\rangle_a$, the phase accumulation is

   $$|\psi\rangle_N = (e^{iN\phi_a}|N\rangle_a|0\rangle + |0\rangle_a|N\rangle_b)/\sqrt{2}.$$

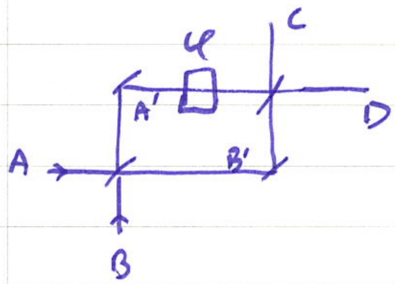
   Problem : inefficient readout protocol

2. squeezing — LIGO (2011)
   Gravitational wave detection
   (~~2023, PRA 47, 5138 ('93~~))
   (2013, Nat. photon. 7, 613 ('13))

A. Super-resolution

1. PBW : photonic de Brogie waves

Theory)　　　　　참조: Science 306, 1330 ('04)



- Quantum optics : deals with application of quantum theory to the optical phenomena.
- Dilemma : wave-particle duality
  → due to incompatibility
- purpose : to find out the values physically measurable quantities
  →
- Quantum mechanics formalism : to incorporate the wave function to the particle nature.
- physical state of a particle : determined by the wave function $\psi(r,t)$.
- Probability of finding the particle in a volume dV :

$$P(r,t)dV = |\psi(r,t)|^2 dV$$

- The equation of motion of the wave function ψ :

$$\hat{H}\psi(r,t) = i\hbar \frac{\partial}{\partial t}\psi(r,t) \quad , \text{Schrödinger Eq.}$$

$$\hat{H} = \hat{T} + \hat{V}$$

morning glory

- Quantum operator:
  - mathematical object to describe physical properties.
  - denoted by hat ^.

$\hat{r}$: position operator (or $\hat{q}$)

$\hat{p}$: momentum operator (or $\frac{\partial}{\partial q}$)

$\hat{H}$: energy operator

$$\hat{q} = q$$

$$\hat{p} = -i\hbar \frac{\partial}{\partial q}$$

$$\hat{H} = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial q^2} + \hat{V}(q) \qquad \left(= \frac{p^2}{2m} + V\right)$$

- One-dimensional Schrödinger Eq:

$$\left(-\frac{\hbar^2}{2m}\nabla^2 + V\right)\psi_n(x,t) = E_n \psi_n(x,t)$$

- For a plane wave,
  - $\psi = \psi_0 e^{i(kx - \omega t)} = e^{\frac{i}{\hbar}(\hbar kx - \hbar\omega t)}$

    $= e^{\frac{i}{\hbar}(px - Et)}$

  - $\frac{\partial \psi}{\partial x} = \frac{i}{\hbar} p \psi$

    $\therefore \hat{p} = -i\hbar \frac{\partial}{\partial x}$

$[\hat{q}, \hat{p}] = i\hbar \hat{I}$: non-commutable.

→ Canonical commutation relation

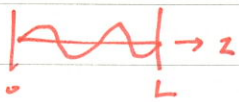
→ Hermitian operators

→ observable quantities

morning glory

- For convenience, non-Hermitian operators are introduced :
  - annihilation $\hat{a}$ $\left(= (2\hbar\omega)^{-1/2} (\omega\hat{q} + i\hat{p}) \right)$
  - creation $\hat{a}^{+}$ $\left(= (2\hbar\omega)^{-1/2} (\omega\hat{q} - i\hat{p}) \right)$
  - $[\hat{a}, \hat{a}^{+}] = 1$

- Electric field satisfying Maxwell Eq.:



  - $E_x(z,t) = \left(\frac{2\omega^2}{V\epsilon_0} \right)^{1/2} q(t) \sin kz$, $k = \frac{\omega}{c}$

- Magnetic field :
  - $B_y(z,t) = \left(\frac{\mu_0\epsilon_0}{k} \right)\left(\frac{2\omega^2}{V\epsilon_0} \right)^{1/2} \hat{p}(t) \cos kz$

    by $\nabla \times E = -\frac{\partial B}{\partial t}$ (Maxwell Eq.)

    $\nabla \times B = \mu_0\epsilon_0 \frac{\partial E}{\partial t}$

  - $H = \frac{1}{2}\int dV \left(\epsilon_0 E^2(r,t) + \frac{1}{\mu_0} B^2(r,t) \right)$

    $= \frac{1}{2}\int dV \left(\epsilon_0 E_x^2(z,t) + \frac{1}{\mu_0} B_y^2(z,t) \right)$

    $= \frac{1}{2}(p^2 + \omega^2 q^2)$

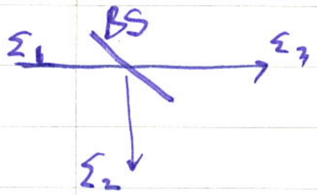
  - $\hat{H} = \frac{1}{2}(\hat{p}^2 + \omega^2 \hat{q}^2)$

- $E_x(z,t) = \mathcal{E}_0 (\hat{a} + \hat{a}^{+}) \sin kz$, $\mathcal{E}_0 = \left(\frac{\hbar\omega}{\epsilon_0 V} \right)^{1/2}$
- $B_y(z,t) = \mathcal{B}_0 \frac{1}{i} (\hat{a} - \hat{a}^{+}) \cos kz$, $\mathcal{B}_0 = \left(\frac{\mu_0}{k} \right)\left(\frac{\epsilon_0\hbar\omega^3}{V} \right)^{1/2}$

- $\hat{H} = \hbar\omega(\hat{a}^{+}\hat{a} + \frac{1}{2})$
- $\hat{a}|n\rangle = \sqrt{n}|n-1\rangle$
- $\hat{a}^{+}|n\rangle = \sqrt{n+1}|n+1\rangle$

morning glory

< BS in quantum operators >



- $\mathcal{E}_2 = r\mathcal{E}_1$,
- $\mathcal{E}_3 = t\mathcal{E}_1$,

  $|r| = |t| = 1/\sqrt{2}$. $\rightarrow |r|^2 + |t|^2 = 1$
- $|\mathcal{E}_1|^2 = |\mathcal{E}_2|^2 + |\mathcal{E}_3|^2$

· Using the annihilation and creation operators,

- $\hat{a}_2 = r\hat{a}_1$
- $\hat{a}_3 = t\hat{a}_1$
- $[\hat{a}_i, \hat{a}_j^\dagger] = \delta_{ij}$, $[\hat{a}_i, \hat{a}_j] = 0 = [\hat{a}_i^\dagger, \hat{a}_j^\dagger]$

  commutation relation in quantum mechanics

· Actually,

$$[\hat{a}_2, \hat{a}_2^\dagger] = |r|^2 [\hat{a}_1, \hat{a}_1^\dagger] = |r|^2$$

$$[\hat{a}_3, \hat{a}_3^\dagger] = |t|^2 [\hat{a}_1, \hat{a}_1^\dagger] = |t|^2$$

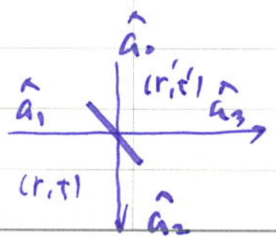
$$[\hat{a}_2, \hat{a}_3^\dagger] = rt^* [\hat{a}_1, \hat{a}_1^\dagger] = rt^* \neq 0$$: violation of the commutation relation.

· With the unused port for the quantized vacuum mode ($\hat{a}_0$):

$\rightarrow \hat{a}_2 = r\hat{a}_1 + t'\hat{a}_0$

$\hat{a}_3 = t\hat{a}_1 + r'\hat{a}_0$

$$\Leftrightarrow \begin{pmatrix} \hat{a}_2 \\ \hat{a}_3 \end{pmatrix} = \begin{pmatrix} t' & r \\ r' & t \end{pmatrix} \begin{pmatrix} \hat{a}_0 \\ \hat{a}_1 \end{pmatrix}$$



$\rightarrow [\hat{a}_2, \hat{a}_2^\dagger] = |r|^2 + |t'|^2$: Energy conserved (reciprocal relation)

$= [r\hat{a}_1 + t'\hat{a}_0 , r^*\hat{a}_1^\dagger + t'^*\hat{a}_0^\dagger]$

$= |r|^2 [\hat{a}_1, \hat{a}_1^\dagger] + rt'^* [\hat{a}_1, \hat{a}_0^\dagger] + t'r^* [\hat{a}_0, \hat{a}_1^\dagger]$

$+ |t'|^2 [\hat{a}_0, \hat{a}_0^\dagger] = |r|^2 + |t|^2$

$\therefore rt'^* + t'r^* = 0 \rightarrow$ Stokes th^m (classical!): $r' = -r$; $t' = t$

morning glory

Thus,

$$\hat{a}_2 = \frac{1}{\sqrt{2}}(\hat{a}_0 + i\hat{a}_1) \quad \& \quad \hat{a}_3 = \frac{1}{\sqrt{2}}(i\hat{a}_0 + \hat{a}_1)$$

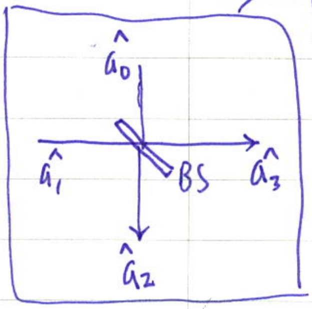
for 50/50 BS.

$$\begin{pmatrix}\hat{a}_2\\ \hat{a}_3\end{pmatrix} = \hat{u}^{+}\begin{pmatrix}\hat{a}_0\\ \hat{a}_1\end{pmatrix}\hat{u}, \quad : \text{BS unitary.}$$

$$\hat{u} = e^{i\frac{\pi}{4}(a_0^{+}a_1 + a_0 a_1^{+})}.$$

· For a single-photon input state $|0\rangle_0 |1\rangle_1$,

→ operator notation : $\hat{a}_1^{+} = (i\hat{a}_2^{+} + \hat{a}_3^{+})/\sqrt{2}$



$$|0\rangle_0 |1\rangle_1 \xrightarrow{BS} \frac{1}{\sqrt{2}}(i\hat{a}_2^{+} + \hat{a}_3^{+})|0\rangle_2 |0\rangle_3$$

$$= \frac{1}{\sqrt{2}}(i|1\rangle_2 |0\rangle_3 + |0\rangle_2 |1\rangle_3).$$

transmitted or reflected with equal probability

↓

☆ Coincidence counts : 0

↓

· anti-bunched single photon (Europhys. Lett. 1, 173 ('86))

"Entanglement" between two output ports.

· For single photons into both input ports, $|1\rangle_0 |1\rangle_1$,

$$\rightarrow |1\rangle_0 |1\rangle_1 \xrightarrow{BS} \frac{1}{2}(\hat{a}_2^{+} + i\hat{a}_3^{+})(i\hat{a}_2^{+} + \hat{a}_3^{+})|0\rangle_2 |0\rangle_3$$

$$= \frac{i}{2}(\hat{a}_2^{+}\hat{a}_2^{+} + \hat{a}_3^{+}\hat{a}_3^{+})|0\rangle_2 |0\rangle_3$$

$$= \frac{i}{2}(|2\rangle_2 |0\rangle + |0\rangle_2 |2\rangle_3).$$

$\left(\hat{a}_2^{+}\hat{a}_3^{+} - \hat{a}_3^{+}\hat{a}_2^{+}\right)$

"entangled"

→ HOM effect

☆ physical basis of no simultaneous counts : destructive interference btwn $|1\rangle_2 |1\rangle_3$ ☆

☆ Limitation: No way to know relative phase btw'n two input photons

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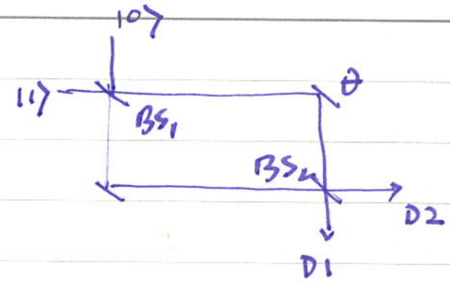


· Unitary operator for the phase shift θ,

→ $e^{i\theta\hat{n}}$,

$\hat{n}$: number operator of the field.

· For a single photon input $|1\rangle_1$:

$$\rightarrow |0\rangle|1\rangle \xrightarrow{BS_1} \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle)_{12}$$

$$\xrightarrow{\theta} \frac{1}{\sqrt{2}}(e^{i\theta}|0\rangle|1\rangle + i|1\rangle|0\rangle)$$

· At BS_2 :

$$|0\rangle|1\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|0\rangle|1\rangle + i|1\rangle|0\rangle)_{12}$$

$$|0\rangle|0\rangle \xrightarrow{BS_2} \frac{1}{\sqrt{2}}(|1\rangle|0\rangle + i|0\rangle|1\rangle)_{12}$$

$$\xrightarrow{BS_2} \frac{1}{2}\left[(e^{i\theta}-1)|0\rangle|1\rangle + i(e^{i\theta}+1)|1\rangle|0\rangle\right].$$

· Probability detected by D_1 :

$$P_{01} = \frac{1}{2}(1-\cos\theta) \quad : \langle 0|\langle 1|0\rangle|1\rangle = (e^{-i\theta}-1)(e^{i\theta}-1)$$

$$P_{10} = \frac{1}{2}(1+\cos\theta) \quad : \langle 1|\langle 0|0\rangle|1\rangle = (e^{-i\theta}+1)(e^{i\theta}+1)$$

· For n photons $|n\rangle_1$, $|n\rangle_0$,

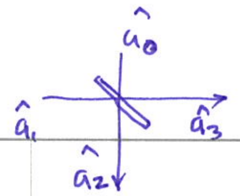
$\rightarrow e^{i\theta} \rightarrow e^{in\theta}$.

$$\therefore P_{01} = \frac{1}{2}(1-\cos n\theta)$$

$$P_{10} = \frac{1}{2}(1+\cos n\theta)$$

※ Requirement : bunched into either path.
↓
∵ destructive interferences!

morning glory



· From $a_1^\dagger = (i a_2^\dagger + a_3^\dagger)/\sqrt{2}$ & $a_0^\dagger = (a_2^\dagger + i a_3^\dagger)/\sqrt{2}$,

$$|2\rangle_0|2\rangle_1 \xrightarrow{BS} \frac{1}{4}(a_2^\dagger + i a_3^\dagger)(i a_2^\dagger + a_3^\dagger)(a_2^\dagger + i a_3^\dagger)(i a_2^\dagger + a_3^\dagger)|0\rangle_2|0\rangle_3$$

$$= \frac{1}{4}\left(i(a_2^\dagger a_2^\dagger + a_3^\dagger a_3^\dagger)(i)(a_2^\dagger a_2^\dagger + a_3^\dagger a_3^\dagger)\right)|0\rangle_2|0\rangle_3$$

$$= -\frac{1}{4}\left(a_2^\dagger a_2^\dagger a_2^\dagger a_2^\dagger + a_3^\dagger a_3^\dagger a_3^\dagger a_3^\dagger + a_2^\dagger a_2^\dagger a_3^\dagger a_3^\dagger \cdot 2\right)|0\rangle_2|0\rangle_3$$

$$= -\frac{1}{4}\left(\underbrace{|4\rangle|0\rangle_{23} + |0\rangle|4\rangle_{23}}_{PBW} + \underbrace{2|2\rangle|2\rangle_{23}}_{\neq 0}\right)$$

· HAM's interpretation! (new understanding)

· What if $\hat{a}_1 = i\hat{a}_0$,

then $a_1^\dagger = i(i a_2^\dagger + a_3^\dagger)/\sqrt{2} = (-a_2^\dagger + i a_3^\dagger)/\sqrt{2}$

$a_0^\dagger = (a_2^\dagger + i a_3^\dagger)/\sqrt{2}$

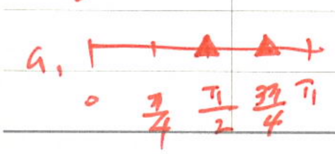
· In addition $\hat{a}_1$ is phase shifted from other $\hat{a}_1$ for multiple photons, i.e., $\frac{\pi}{2N}$, where N is photons in each ~~m~~ mode.

· Thus, for $N = 2$, $a_1^\dagger a_1^\dagger \longrightarrow a_1^\dagger e^{i\frac{\pi}{4}} e_1^\dagger$

$a_0^\dagger a_0^\dagger \longrightarrow a_0^\dagger e^{i\frac{\pi}{4}} e_0^\dagger$

· Then,

$$|2\rangle_0|2\rangle_1 \xrightarrow{BS} \frac{1}{4}\Big(-a_2^\dagger + i a_3^\dagger)(i a_2^\dagger + i a_3^\dagger) e^{i\frac{\pi}{4}}(-a_2^\dagger + i a_3^\dagger)(i a_2^\dagger + i a_3^\dagger) e^{i\frac{\pi}{4}}|0\rangle_2|0\rangle_3$$



$$= \frac{1}{4}(i)(-i a_2^\dagger a_2^\dagger - a_3^\dagger a_3^\dagger)(\cancel{\ \ })(-i a_2^\dagger a_2^\dagger - a_3^\dagger a_3^\dagger)|0\rangle_2|0\rangle_3$$

$$= -\frac{i}{4}(i a_2^\dagger a_2^\dagger + a_3^\dagger a_3^\dagger)(a_2^\dagger a_2^\dagger - i a_3^\dagger a_3^\dagger)|0\rangle_2|0\rangle_3$$

$$= \frac{-i}{4}\left(a_2^\dagger a_2^\dagger a_2^\dagger a_2^\dagger + a_3^\dagger a_3^\dagger a_3^\dagger a_3^\dagger\right)|0\rangle_2|0\rangle_3 = \frac{-i}{4}\left(|4\rangle_2|0\rangle_3 + |0\rangle_2|4\rangle_3\right)$$

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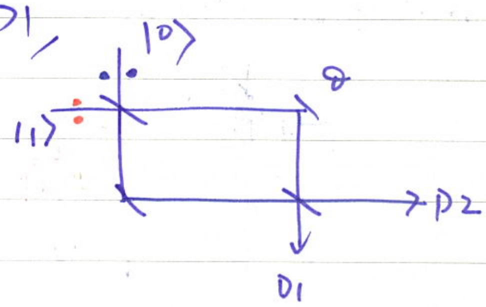
· Probability detected by D1, via $e^{i\theta n}$,

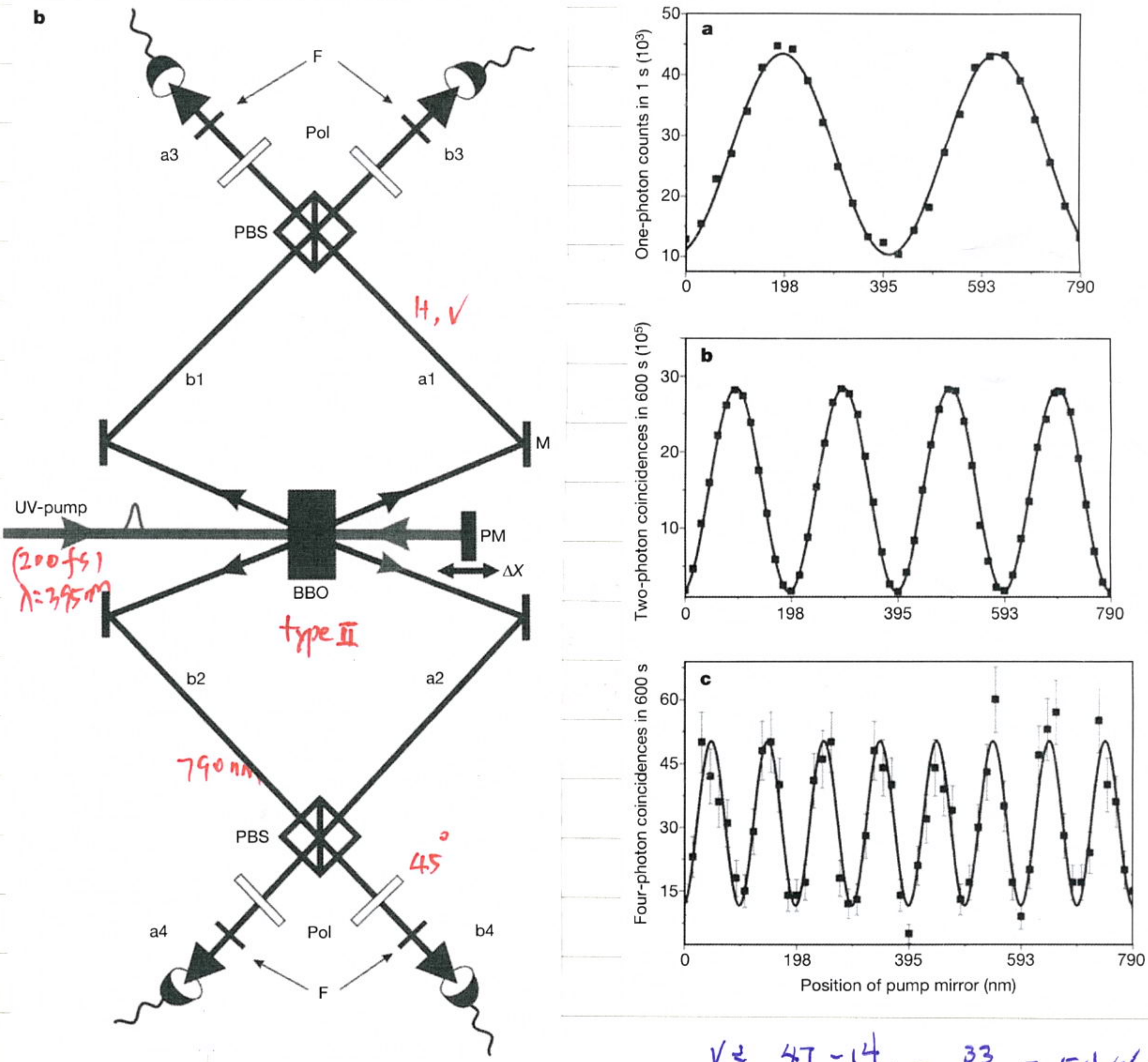


· $P_{01} = \frac{1}{16}(1 - \cos 4\theta)$,

$P_{10} = \frac{1}{16}(1 + \cos 4\theta)$.

< Photonic de Broglie Waves >

Using entangled photon pairs

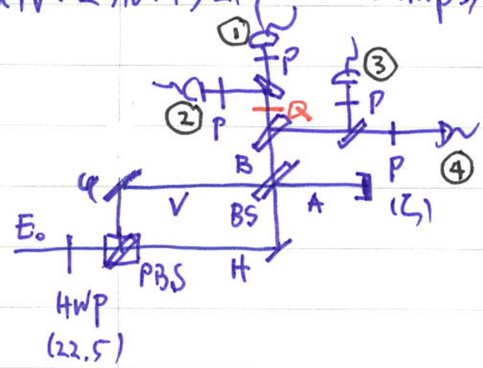


$V = \frac{47-14}{47+14} = \frac{33}{61} = 54(\%)$

출처: Nature 429, 158 (04) · Superresolution but no supersensitivity

morning glory

출처: HAM
arXiv: 2310.13217 (https// arxiv.org → search with 2310.13217)



Q : Quarter-wave plate

H → H
V → iV
} for 0° degree rotation

- HWP (Half-wave plate)
  H → H + V (superposition)

- B port : $E_B = \frac{E_0}{2}(\hat{H} - \hat{V}e^{i\varphi})$, $\hat{H}$ & $\hat{V}$: polarization unit vectors.

· reflected image (π)

$$E_4 = \frac{i}{2}E_B = \frac{iE_0}{4}(-\hat{H} - \hat{V}e^{i\varphi})$$

$$E_3 = -\frac{1}{2}E_B = -\frac{E_0}{4}(\hat{H} - \hat{V}e^{i\varphi})$$

$$E_1^Q = \frac{1}{2}E_B' = \frac{E_0}{4}(\hat{H} - i\hat{V}e^{i\varphi})$$

π shift

$$E_2^Q = \frac{i}{2}E_B' = \frac{iE_0}{4}(-\hat{H} - i\hat{V}e^{i\varphi})$$

By 45° polarizer, $\hat{H} \to H\cos\zeta$ & $\hat{V} \to \hat{V}\sin\zeta$

$$E_1^Q = \frac{E_0}{4}(\hat{H}\cos\zeta - i\hat{V}\sin\zeta e^{i\varphi})\hat{p}$$

$$E_2^Q = \frac{-iE_0}{4}(\hat{H}\cos\zeta + i\hat{V}\sin\zeta e^{i\varphi})\hat{p}$$

$$E_3 = -\frac{E_0}{4}(\hat{H}\cos\zeta - \hat{V}\sin\zeta e^{i\varphi})\hat{p}$$

$$E_4 = -\frac{iE_0}{4}(\hat{H}\cos\zeta + \hat{V}\sin\zeta e^{i\varphi})\hat{p}$$

morning glory

· Thus, intensities of the output beams :

$$I_1^Q = \frac{I_0}{16}(H\cos\zeta - iV\sin\zeta e^{i\varphi})(H\cos\zeta + iV\sin\zeta e^{-i\varphi})$$

$$= \frac{I_0}{16}(\cos^2\zeta + \sin^2\zeta + i\cos\zeta\sin\zeta(-e^{i\varphi} + e^{-i\varphi}))$$

$$= \frac{I_0}{16}(1 + \sin 2\zeta \sin\varphi)$$

$$I_2^Q = \frac{I_0}{16}(H\cos\zeta + iV\sin\zeta e^{i\varphi})(H\cos\zeta - iV\sin\zeta e^{-i\varphi})$$

$$= \frac{I_0}{16}(\cos^2\zeta + \sin^2\zeta - i\cos\zeta\sin\zeta(e^{-i\varphi} - e^{i\varphi}))$$

$$= \frac{I_0}{16}(1 - \sin 2\zeta \sin\varphi)$$

$$I_3 = \frac{I_0}{16}(H\cos\zeta - \hat{V}\sin\zeta e^{i\varphi})(H\cos\zeta - \hat{V}\sin\zeta e^{-i\varphi})$$

$$= \frac{I_0}{16}(\cos^2\zeta + \sin^2\zeta - \cos\zeta\sin\zeta(e^{i\varphi} + e^{-i\varphi}))$$

$$= \frac{I_0}{16}(1 - \sin 2\zeta \cos\varphi)$$

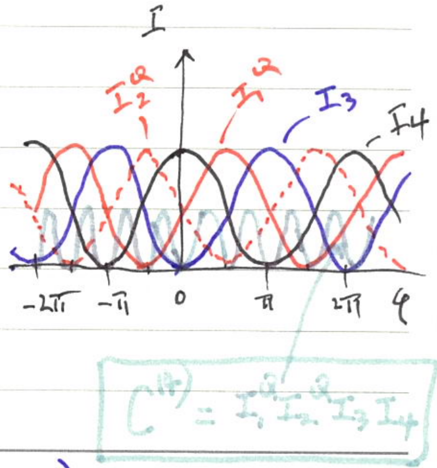
$$I_4 = \frac{I_0}{16}(1 + \sin 2\zeta \cos\varphi)$$

For $\zeta = \frac{\pi}{4}$, (45° polarizer)

$$\sin 2\zeta = 1.$$

Thus,

$$I_1^Q = \frac{I_0}{16}(1 + \sin\varphi)$$
$$I_2^Q = \frac{I_0}{16}(1 - \sin\varphi)$$
$$I_3 = \frac{I_0}{16}(1 - \cos\varphi)$$
$$I_4 = \frac{I_0}{16}(1 + \cos\varphi)$$



· Super-resolution :

· Super-sensitivity ?! ($\eta = 1$) • $C^{(4)} \propto (1 - \cos 4\varphi)$

$V > 1/2$

morning glory