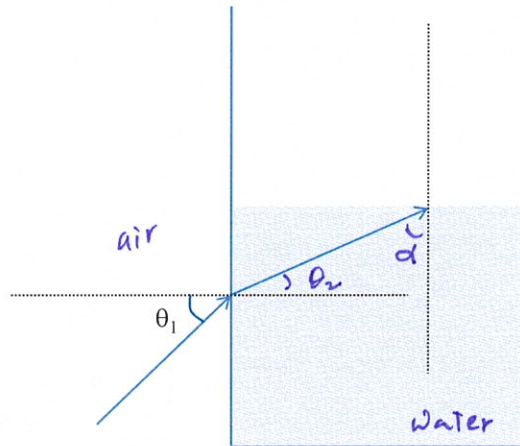


A laser beam is incident on the side of a rectangular fish tank with angle  $\theta_1$  from the normal to the glass surface. The beam enters the water and strikes the surface of the water. For what range of angles  $\theta_1$ , does the beam undergo total internal reflection at the water-air interface? The refractive index of air and water is 1.0 and 1.3, respectively.



Sol) By Snell's law,

$$\textcircled{1} \quad n_1 \sin \theta_1 = n_2 \sin \theta_2 \quad \text{at the interface of the tank}$$

$$\textcircled{2} \quad n_2 \sin \alpha = n_1 \sin \theta (90^\circ) = n_1 = 1 \quad \text{at the interface of water}$$

$$\begin{aligned} \theta_2 + \alpha &= 90^\circ & \rightarrow & n_2 \sin \alpha = n_2 \sin \left( \frac{\pi}{2} - \theta_2 \right) = n_2 \cos \theta_2 \\ & & \rightarrow & n_2 \sin \theta_2 = n_2 \sin \left( \frac{\pi}{2} - \alpha \right) = n_2 \cos \alpha \\ & & & = n_2 \sqrt{1 - \sin^2 \alpha} \\ & & & = \sqrt{n_2^2 - 1} \end{aligned}$$

$$\therefore n_1 \sin \theta_1 = n_2 \sin \theta_2 = \sqrt{n_2^2 - 1}$$

$$\rightarrow \sin \theta_1 = \frac{\sqrt{n_2^2 - 1}}{n_1} = \sqrt{n_2^2 - 1} = 0.83$$

For the total internal reflection,

$$\underline{\underline{\sin \theta_1 < 0.83}} \quad \text{or} \quad \theta_1 < 56.1^\circ$$